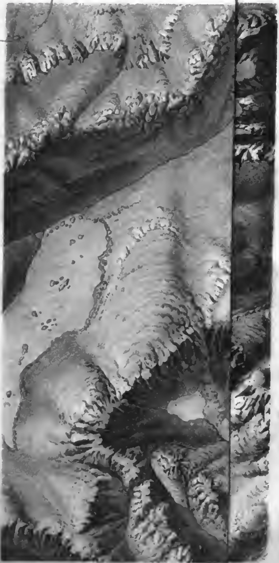
PRACTICAL GEODESY.



PRACTICAL GEODESY:

COMPRISING

CHAIN SURVEYING AND THE USE OF SURVEYING INSTRUMENTS;

TOGETHER WITH

TRIGONOMETRICAL, COLONIAL, MINING, AND MARITIME SURVEYING;

ALSO

LEVELLING AND HILL DRAWING;

AND

A DESCRIPTION OF THE METHODS OF DETERMINING

LATITUDES AND LONGITUDES.

ADAPTED TO THE USE OF SURVEYORS, AND OF STUDENTS IN CIVIL, MILITARY, AND NAVAL ENGINEERING.

BY

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TO

HIS GRACE

THE DUKE OF BUCCLEUCH,

PRESIDENT OF THE COLLEGE FOR CIVIL ENGINEERS,

&c., &c., &c.,

WHOSE UNCEASING LABOURS TO AMELIORATE
THE CONDITION OF HIS COUNTRY
BY THE PRACTICAL APPLICATION OF SCIENCE TO ITS
INTERNAL IMPROVEMENT
HAVE BEEN ATTENDED WITH MOST IMPORTANT RESULTS,

AND WHO

EFFECTUALLY PROMOTES THE ARTS OF PEACE
BY THE ENCOURAGEMENT OF SCIENTIFIC EDUCATION,

THIS TREATISE

IS, BY PERMISSION,

DEDICATED WITH THE GREATEST RESPECT,

BY HIS GRACE'S
MOST OBEDIENT
HUMBLE SERVANT,

THE AUTHOR.

PREFACE.

I HAVE employed the term Geodesy, as one well suited by its comprehensive meaning to embrace the various subjects treated of in the present work; and, by adding the word 'Practical,' I have designed to indicate that actual practice in the field is the especial object I have had in view. This has not been lost sight of throughout the work, and the subject has been so arranged as to enable the student, at the outset, to commence a simple survey, minute practical directions being given to guide him in each new process, or to prepare him to surmount, by his own judgment, difficulties which may arise under any new aspect.

Proceeding through the elementary processes, the student is brought, by regular gradations, to understand the principles, and to appreciate the bearings of the more perfect methods which the demands of an advanced state of civilization render it necessary to employ, in order to ensure that degree of accuracy which is now deemed essential.

In the Chapter on Trigonometrical Surveying, I have taken the opportunity of making the student acquainted, by a description of some of the interesting trigonometrical operations of this and other countries, with the highly scientific processes

required for the successful performance of an extensive survey. It is true that in Great Britain the trigonometrical operations are nearly completed; but we find that, as every year increases the civilization and adds to the wealth of our older Colonies, similar operations, conducted with an approach to equal accuracy, are called for in those portions of the British Empire.

It has been deemed also that, with the tide of emigration flowing so freely towards the New Colonies, a treatise on Surveying would have been incomplete without a description of the method of surveying especially applicable to New Colonies.

The subject of the measurement of base lines, naturally called for an account of standard measures, a subject of high interest at all times, but having especial claims to the attention of the surveyor now that the Legislature is again on the eve of enacting regulations on the subject; and when we find it recommended by the Commissioners appointed to report on the formation of new standards, that copies of the Parliamentary standards should be deposited in every town of more than 10,000 inhabitants, and be placed for reference under the care of an intelligent surveyor.

An exposition of the insufficiency of the instruments in general use for mining surveys has been deemed of importance, for it appears strange that mining surveys should, to this day, be performed with instruments which have long been laid aside as too inaccurate for surveys above ground, although

the latter seldom present difficulties of so serious a character as those met with in subterraneous operations. Among other improvements, the introduction of a theodolite, so constructed that its axis can be made, with perfect certainty, to occupy in succession the locus of the light which constituted the object previously observed, is in itself of great value. The accurate results sought in driving "headings" or "adits," for railway tunnels, first led to the general introduction of improved methods; and here it may be observed, that railway surveys and railway works have, among other changes, been the means of introducing very great improvements in Levelling, and most minute accuracy in the performance of surveys for detailed plans.

In the Chapter on Maritime Surveying, particular care has been taken to convey practical information on those details, which, although they may be familiar to the mariner, are novel to the engineer and land-surveyor, on whom the duty not unfrequently devolves of conducting surveys of harbours and of coasts connected with interior triangulation.

I have in the course of the work acknowledged, by references at the foot of the page, the sources from whence I have derived any portion of the contents. But I am desirous especially to point out the valuable assistance which I have received in the Chapter on Latitude and Longitude from Sir J. F. W. Herschel's *Treatise on Astronomy*. It is acknowledged that the subject of longitude is one

not generally understood, even by many among those whose professional pursuits render it necessary that they should frequently solve problems in connexion with it; which solutions they perform by reference to rules, without any knowledge of the principles on which the rules are founded. One reason of the little success with which this branch of science has been cultivated, is no doubt to be found in the obscurity, or abstract profundity, of the many treatises that have been published; but any one who will peruse with common care Sir John Herschel's luminous work on the subject cannot fail to become master of its general principles.

Finally, I would express a hope that the present volume, by affording frequent illustrations of the application of mathematical knowledge to purposes eminently practical, may be instrumental in encouraging, amid the varied occupations of an active life, the pursuit of that science, which, if cultivated with judgment, leads to most beneficial results, and is capable of promoting the highest intellectual enjoyments.

London, April, 1842.

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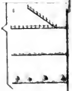

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PRACTICAL GEODESY.

CHAPTER I.

SURVEYING WITH THE CHAIN.

To construct a map of a district of country consists in making on paper a figure similar to the ground, imitating, on a smaller scale, as nearly as possible all the prominent objects on the surface.

A topographical map or plan differs from a geographical map in this particular, that the first is intended to represent all or nearly all the details that appear on the surface of the country; the second embraces a comparatively great extent of country and aims only at determining the relative position of the principal points and most important objects.

Surveying consists in measuring, with a view to subsequent delineation on a map or plan, the boundaries and forms of natural and artificial objects on the surface of the ground, ascertaining and fixing their relative position, and finding the superficial content or area of the whole as well as each part.

Land is surveyed either by means of a chain only, or by combining it with a theodolite or other angular instrument. We commence by describing the first of these methods, as being the most elementary; and well adapted for the measurement of a small extent of surface.

The chain is a linear measure constructed of any given arbitrary length, divided by links into a stated number of units of extension. A chain of 100 feet, divided into 100

links, is perhaps the most convenient for general use, being equally applicable to the purposes of the engineer and land surveyor, and combining readily horizontal with vertical distances, the latter of which are uniformly measured in feet, whatever unit may otherwise be adopted for the former. When the sole object is to obtain the acreage, Gunter's chain, equal in length to 66 feet, is the most convenient, because of the facility it affords for computation: as 10 square chains are equal to 1 acre, and as the chain is divided into 100 links, the contents, expressed in chains and links, are converted into acres and decimals of an acre, by simply dividing by 10. (See *Computation of Areas*, page 29.)

To guide the eye in counting the number of feet or links, brass marks are fastened at every tenth foot or link, and distinguished from each other by notches, each varying in number according to their position with respect to the extremity of the chain; so that the surveyor can, by simple inspection, readily read any number of feet required.

Accompanying the chain are 10 arrows which are used in succession to mark the end of the chain in measuring a line.

The chain is used by two persons, one of whom is called the leader, the other the follower. The point from which the measurement is to commence, as also the direction of the line to be measured, being determined, the leader, who has been supplied with the 10 arrows, stretches the chain in the required direction, while the follower keeps one end of it at the starting point. An arrow is then thrust perpendicularly into the ground by the leader, at the point where the chain terminates; he then proceeds onward drawing the chain after him, and repeats the same operation throughout the length of the line, the arrow last put down serving always as the mark to which the follower is to bring his end of the chain as a new station of departure.

The arrows are taken up by the follower as he advances, and when the 10 arrows have thus changed hands, they are

all returned to the leader to be used again. In this manner the arrows are changed from one to the other at every 10 chains' length, till the whole length of the line is measured, care being had to enter every such change in the field-book. At the end of the line, the number of changes, added to the number of arrows in the follower's hand, and to the number of feet extending from the last arrow put down to the extremity of the line, gives the entire length measured.

When long distances are to be measured, a very convenient check, in the counting of the number of chains, is obtained by using 10 supplementary arrows, distinguished from the common ones by brass or other marks, these arrows to be put down in succession at every tenth chain. In this case, the number of arrows each indicating 1 chain, is reduced to 9; and as each one of the supplementary arrows represents the measure of 10 chains' length, when the 10 supplementary arrows shall have passed into the hands of the follower, a distance of 100 chains' length will have been measured. Another advantage resulting from this arrangement is that it avoids the inconvenience occasioned by the follower having no arrow to measure from, after he has given his 10 arrows to the leader.

In the operation just described, care is necessary, first, to ensure that the line gone over is a straight line; secondly, that the ends of the chain have been made to coincide as accurately as possible with the arrows placed to mark each extremity. A deviation from a straight line causes the apparent length measured to be greater than the trne length: this follows from Euclid's 20th proposition of the first book, which proves that any two sides of a triangle are together greater than the third. Also, the frequent repetition of errors in the coincidence of the extremities of the chain with the arrows, may render of great effect deviations which, viewed singly, would seem inconsiderable.

We would therefore recommend a young surveyor, when employing as chainmen labourers who have been

unused to the work, to cause them to measure a certain distance, say half a mile or a mile, on a level road, several times, until they shall have learned, by a careful attention to the directions above given, to bring very nearly the same result at each measurement. This preliminary caution will save the surveyor much subsequent loss of time, first, by training his assistants to their work; and secondly, (which is of more importance,) by impressing them with the necessity of a very careful admeasurement as an element in the successful performance of the survey.

The length of the chain should from time to time be tested by a careful comparison with a standard chain kept for the purpose. To facilitate and expedite this comparison, two strong pickets may be driven firmly on a level portion of ground, at a distance of 100 feet from each other; and, the standard chain being stretched from one to the other, notches are cut or nails driven marking the exact length of the 100 feet. The coping of a horizontal wall is also well suited to receive such marks. They are thus rendered permanent, and every day a comparison of the working chain may be made with the standard length without loss of time. This precaution is indispensable; for the chain, being composed of numerous pliable links joined together by three small unwelded rings which give it flexibility, is from its construction constantly liable on the one hand to expand at the joints, and on the other to have the links bent when dragged over rough surfaces. Of such importance is this examination held by scientific and practical men that the Commissioners for the restoration of the Standards of Weight and Measure recommend in their report, dated December 21, 1841, that "no person shall be admitted to give evidence in any court of justice of having measured land, after the passing of the contemplated Act, with any other than a stamped (standard) measure, or a measure which has been compared, on each day on which any part of the measurement has been made, with a stamped (standard)

measure." The chain used for measuring may be left unaltered, if it be about half an inch longer than the standard, as it can never be stretched perfectly straight, but adapts itself by its weight to the small inequalities on the surface. The chain also, when used in wet weather, becomes shorter, in conseqnence of the insertion of dirt between the rings. If the excess be great, the length must be diminished by the removal from each extremity of one or more of the rings. The same correction, whatever it be, should be made equally at both extremities, in order that the middle point of the chain may be in its true position, in which case the error, subdivided among the remaining parts, will be trifling, and not worth being taken into account. If, on the contrary, the working chain be found too short, this will arise from the links being bent: consequently by straightening them, the correction is effected.

In measuring an estate, a parish, or any comparatively small portion of land, the surface may be supposed to be divided into a system of arbitrary geometrical figures bounded by right lines, either inscribed within or circumscribing the area to be measured; but so disposed that they shall pass near all objects included in the snrvey, and serve to determine their positions and forms. The object of this imaginary division is to facilitate the measurement of all irregular boundaries, which, if they were traced independently of these auxiliary lines, into all their windings, would lead to a great consumption of time in the operation and to inaccuracy in the results, owing to the number of mutually dependant angles to be measured. After having divided the surface into a number of geometrical figures bounded by right lines, the sides of these figures are used as bases from which the irregular bonndaries and other objects are measured by means of shorter lines at right angles, termed offsets.

These offsets, when short, are measured with an offset staff 10 feet in length; with a second chain, or, in preference,

a measuring tape, when the offsets are too long for the staff to measure them conveniently. The limit to the length of these offsets is fixed in a great measure by the degree of accuracy aimed at in the survey, as well as by the scale to which the plan is to be drawn; in general it is not advisable to make use of offsets more than about 100 feet in length.

Offsets, with few exceptions, are measured at right angles to the main line: the length of the offset being once determined, the position of the object referred to is fixed with reference to the main line. These right angles must of course be set off correctly. The instrument formerly employed for setting out perpendicular lines was the cross staff. It consists of sights fixed at right angles to the extremities of the arms of a cross, and adapted to the top of a short staff, which being thrust into the ground with two of the sights placed in the direction of the main line, points out the perpendicular required, by means of the other two sights. This instrument is simple and convenient; it has been, however, generally superseded by the optical square, a small circular box, of about 2 inches in diameter, which marks a right angle with accuracy and expedition.

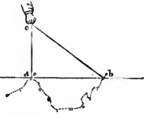
This box contains two glasses fixed at an angle of 45° to each other; one of them acting as a mirror, the other half-silvered to admit of direct vision of one object and of reflected vision of the second placed at right angles to a line passing from the observer to the first; the image of which second object is reflected from the first mirror. This instrument is constructed on the principle that the angle made by the first and last directions of a ray of light which has suffered two reflections in one plane is equal to twice the angle of inclination of the reflecting surfaces. This theorem will be fully investigated and explained, when we come to treat of the sextant, of which the optical square is a particular application.

A line may be laid out at right angles to another, from

any point in it, by simply standing with this instrument over the given point, and looking through it along the line, having an assistant to go with a mark or ranging rod in the direction in which the perpendicular is required, and signing to him to move to the right or the left, until his rod is seen by reflection to coincide with a staff fixed on the line along which the observer is looking. When the coincidence takes place, the rod is fixed in the ground.

If, instead of erecting a perpendicular from a given line at a given point, it be required to find on a line the point of intersection of a perpendicular from a fixed object, as a house, a tree, &c., the observer himself must move along the line until the image of the object appears, as before, in the direction of the line, and the place where he then stands marks the spot where the perpendicular would fall.

The following is a correct method of setting out a perpendicular with the chain only. Let *a b* be the line on which it is required to erect a perpendicular from the point *a*. Fix an arrow in the ground at *a*, through the ring of the chain denoting 20 feet, and measure 40 feet on *a b*. At *b* fix the *extreme* end of the chain; then, holding the brass ring denoting the 50 feet, or the centre of the chain, draw the chain tightly in the direction *c*, the sides of the triangle will then be in the proportion of 30, 40, 50, and consequently *b a c* will be a right angle.



This method of erecting a perpendicular is frequently useful, but it is evidently too laborious to be applicable to the purpose of offsets.

SURFACE TO BE DIVIDED INTO TRIANGLES.

In making a survey with the chain only, we are confined to one and the simplest geometrical figure,—namely, the triangle: for, of all plane geometrical figures, it is the only one of which the form cannot be altered, if the sides remain constant. That the triangle possesses this property is evident from the theorem, (Euclid, 7, I.) which proves that, "Upon the same base, and on the same side of it, there cannot be two triangles that have their sides which are terminated at one extremity of the base equal to one another, and likewise those which are terminated in the other extremity, equal to one another."

The surface to be measured is therefore divided into a series of imaginary triangles; and, in this division, it must be borne in mind that the triangles are to be as large, with reference to the whole surface to be measured, as is consistent with the nature of the ground; for, by such an arrangement, we are acting on this important principle in all surveying operations, that it is well always to work from a whole to the parts, and rarely from the parts to a whole. By the first method errors are subdivided, and time and labour economized; by the second, the errors inseparable from all operations that do not deal with abstract quantities are increased as each step in the work advances.

The sides of these triangles are first measured; and, as a necessary check on this first part of the work, a straight line is in addition measured from one of the vertices to a point in or near the middle of the opposite side. This fourth line is called a tie-line or proof-line, and is an efficient means of detecting errors if any have been committed in the admeasurement of the sides of the triangle. This fourth measurement is made in accordance with a maxim which ought invariably to be acted upon in all surveying operations, whether limited or extensive, simple or complex; namely,

that where accuracy is aimed at, the dimensions of the main lines, and the positions of the most important objects, should be ascertained or tested by at least two processes independent one of the other.

Within the larger triangles, as many tie-lines and smaller triangles are to be measured as may be necessary to determine the position of all the objects embraced in the survey. The directions of the lines forming the sides of these secondary triangles are so selected or disposed that they shall connect, and pass close to, as many objects as possible, so that the offsets to be measured from them may be as short, and as few in number, as practicable.

If the sides of these secondary triangles be in any case so distant from the objects whose positions are to be determined as to require a length of offset greater than the proposed limit of about 100 feet, it then becomes advisable to construct, either on the whole or a part of the side of the triangle as a base, a smaller offset triangle with the sides so disposed that they shall either embrace, or pass very near to, the objects to be measured by their intervention.

The disposition and general combination of these triangles demanding care and judgment, it is customary, previous to commencing any measurement, to walk over the ground for the purpose of obtaining a general knowledge of the surface, and of the relative positions of the most conspicuous objects. The acquisition of this knowledge, depending on the *coup d'œil*, is much assisted by an eye-sketch drawn with rapidity, and showing some of the principal roads, streams, churches, &c. This hand-sketch is not to be drawn to any scale; and its object is attained if it simply bear a general resemblance to a plan of the ground, as it will thereby assist the memory in the distribution of the surface into triangles.

The sides of the larger triangles are to pass as close as possible to the external boundaries to be surveyed; the triangles should moreover be made to approach, as nearly as

practicable, to the form of equilateral, avoiding with care very acute or obtuse angles, because the farther the form of the triangle is removed from the equilateral, the greater will be the alteration in the form of the figure and in its area, should any error have been committed in the measurement of any one of the sides. (See note, Chapter III.,—theorem on *Well-conditioned Triangles.*)

The triangles having thus been disposed to the greatest advantage, marks or pickets are placed on the ground at each vertex of the triangles. Their general form and position is then noted on the hand-sketch previously made, and distinctive letters are written on the diagram at each point of intersection. This arrangement admits of easy reference in the field-book, or on the ground, to any triangle or part of a triangle.

The points of intersection of all straight lines, as well as the vertices of the triangles, are always points measured to or from: they are called station-points or stations, and the lines connecting them station-lines, thereby distinguishing them from the simple offset lines.

Field-Book.

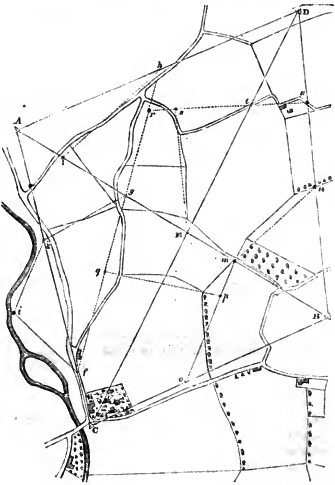
The hand-sketch or rough diagram is usually made in the field-book, that is, a book in which every minute step of the operations gone through in the survey is to be entered with precision at the time. The entries in the field-book should be made with an indelible pencil, or written in ink in the field; but the first is to be preferred, as it is difficult to make legible entries in ink in wet weather. The field-book should be paged for convenience of reference.

The field-book is ruled into three columns; in the middle one are set down the distances on the station-line, at which any mark, offset, or other observation is made; and in the right or left-hand column are entered the offsets and observations made on the right or left-hand respectively of the station-line.

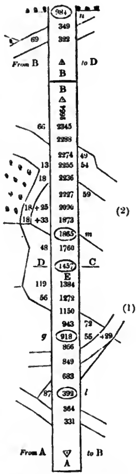
It must be borne in mind that the middle column in the field-book represents the position of, or rather the station-line itself. If the station-line, therefore, should be crossed on the ground by a fence, or any bouudary meeting it obliquely, its representation or type in the field-book must not be made to pass obliquely across the middle column, but must arrive at one side of the column and leave it on the other, at points precisely opposite, as it would do were the middle column merely of the thickness of a line. Inattention in this particular causes much confusion in the relative position of offsets.

It is a universal rule, and of advantage for the sake of perspicuity, to begin the entries in the field-book at the bottom of the leaf; the reason is evident, inasmuch as it places the field-book in the same position as the station-line with respect to the surveyor, who keeps his face directed towards the distant station. The crossings of the fences,

roads, streams, &c., and the corners of fields, and other remarkable turns in the boundaries to which offsets are taken, are to be shown by joining lines in a manner somewhat similar to the form which they assume on the ground. It is a too common error with the inexperienced surveyor to neglect this approximation to the real forms of the



objects, as also to make his entries faintly, and with a careless hand. It cannot be too strongly impressed on the surveyor that the work which he is called upon to perform depends for its accuracy in a very great measure on the order, system, and neatness bestowed on all the steps, whether of delineation or of measurement. Proper attention in keeping the field-book saves much time in plotting, and guards against the errors unavoidably arising from reference to a confused field-book. Moreover, care bestowed in the first essays will amply reward the surveyor, by giving accuracy of eye, freedom, and steadiness of hand, qualities indispensable to his success.



Form for Field Book.

On commencing the measurement of a station-line, the letter corresponding to the starting point in the rough diagram is entered first at the bottom of the middle column, and on each side are written the letters marking the extremities of the line, thus: from A to B, A being the starting point. In the same manner, on arriving at the end of the line corresponding to the point B, the letter B is written in the middle column above the closing distance,

and above the letter a line is drawn across the middle column to denote that the line terminates in that point. In the form of field-book given in the preceding page are entered the observations for the line A B on the annexed diagram. The numbers placed within a vinculum on the right hand side of the page, indicate the changes of the ten arrows. If the line be a short one, and a staff at either extremity be seen from every part of it, it may be ranged by the eye. If the line be so long, or on such uneven ground that the staff, fixed at its extremity, be occasionally lost sight of, the line must be ranged with a telescope from a commanding position, care being taken that the axis of the telescope be truly in the line: pickets or ranging rods are with its assistance placed at convenient distances in the right line. Advancing along it with the chain, the distance of the crossing of every fence or natural boundary, as also of all points from which offsets are taken, is noted in the field-book, together with the lengths of these offsets measured to the right or left. At every station, that is, any of the points determined by the intersection of the sides of the triangles or of tie-lines with those sides, a picket is driven, so that the precise spot may be readily found again in subsequent parts of the operation. The number of feet written in the field-book as corresponding to the distance of such station, is encircled by a line to distinguish it from other points, and by the side of the circle the letter corresponding to the point in the diagram is written. This process is continued until the measurement has extended to the extremity of the line, of which the entire length is written down in larger characters than the rest; and, as an additional distinction, written in a line parallel with the vertical sides of the field-book. When it is not convenient to measure exactly *from* any of the marks left for the tie-lines, the place measured from may be described as being so many feet from one station towards another; and where a station-mark is not measured *to* pre-

cisely, the exact place at which the measurement is stopped is ascertained by writing "turn to the right (or left)" so many feet towards such a station, it being always understood that these auxiliary distances are measured along the station-line.

In this manner, the sides of all the triangles are measured in succession, and their dimensions with the additional assistance of offsets, give the means of ascertaining all boundaries, external and internal, positions of houses, &c., and of finding the area of the whole and of every part by direct computation from the field-book. But to obtain the contents of each inclosure by computation, would be a process very laborious and generally unnecessary: the contents of the whole should be ascertained by computation from the sides of the large circumscribing triangles; the areas of the inclosures may be afterwards obtained by measurement from the plan, their accuracy being tested by a comparison of the sum of the areas of each inclosure with the area comprised within the exterior boundary, as obtained from direct computation.

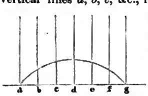
We now come, in connexion with this question of areas or superficial contents, to the consideration of an important principle, namely, the reduction of the lines measured over steep slopes to the horizontal plane.

In the first place, having to lay down on a plan or flat surface boundaries and lines at different inclinations, in order to avoid distortion in the outline, and to bring all the details duly within the triangular framework, it is absolutely necessary that we refer to, or project all lines and points upon a plane. The plane adopted to receive this common projection is the horizontal plane. It is not, therefore, the actual surface that we have to protract, but the diminished quantity that would result, had the whole been reduced to a horizontal base.

In the second place, this distinction, which is indispensable for the purpose of laying down a plan of the surface,

is supported, when the qnestion is viewed nnder its social aspect, by the obvious principle that since plants shoot up vertically, the vegetable produce (with the exception, perhaps, of grasses), on a swelling eminence, does not exceed in quantity what would have grown from its base.

A diagram makes this proposition evident: for let the vertical lines *a*, *b*, *c*, &c., represent the position of plants growing as closely as possible, or as is judged advantageous, from the horizontal surface *a g;* it is manifest that if a curved line be drawn resting on the base *a g*, and



representing the inclined surface of the soil, the same number of plants only can grow in the vertical direction which plants tend to assume. The arrangement, therefore, as a matter of graphic necessity, is not inconsistent with the order followed by nature.

All sloping or hypotenusal distances are, consequently, reduced to their horizontal lengths. When the lines are long, and the slopes much varied and considerably inclined, this reduction ought to be made by calculation, or at least by reference to tables of reduction usually engraved on the vertical arcs of angular instruments, which, while they show on one side the angle of elevation or depression, give on the other the number of units per hundred that have to be deducted to reduce the hypotenuse to its corresponding horizontal length. This subject we are not at present discussing; and in small surveys, especially those made with the chain only, an allowance or reductiou is generally made in the field by construction or estimation as the measurement proceeds.

If the slope be not very steep, the reduction is accomplished by holding the lower end of the chain above the ground, as nearly horizontal as can be judged by the eye, allowing a pointed plummet to hang from the hand that

holds the chain, in order to point to where the arrow shall be placed. If the slope be steep, one half or one quarter of the chain is used, as being more easily brought to a horizontal position; and on precipitous banks the offset staff or measuring tape is substituted, as giving more correct results, with greater expedition. It may be observed, that when the chain is thus held suspended, it cannot be straightened, its links describing the catenary curve; but as a compensation for the shortening of the chain caused by the bend, it is found that the pull at each end of the chain to diminish the curvature caused by its weight, tends to open each unwelded elastic ring, and thus adds very sensibly to the length which it would have, if laid on the ground. Nevertheless, the bending of the chain is an element of inaccuracy in this process, which is further made erroneous by the difficulty of ascertaining without lateral or longitudinal error the exact point vertically beneath the suspended extremity of the chain; and by the unavoidable deviation from the horizontal line, when it has to be estimated by the eye.

The surveyor who does not use an angular instrument for the purpose of ascertaining the reduction required, learns by habit to estimate and make at each chain's length on the ground an approximate reduction; in which operation he will be much assisted by the annexed table, which may be copied on the first leaf of the field-book, and the principal elements of which are learned after some references and practical applications. The inclination in such cases is, of course, estimated solely by the eye; and we describe this method, not to recommend it, but because it will aid the surveyor in approaching to accuracy in exceptional instances where he may not have the assistance of an angular instrument. The reductions are purposely made approximate in the table, in order not to distract the attention by fractional quantities in the application of a process, in itself only an approximation.

Reduction in feet upon each length of 100 feet for the following Angles of Inclination, or for the following Rates of Inclination, expressed in terms of the Horizontal Base and Vertical Height.

ANGLE.	Corresponding Rate of Inclination. (Approximative.)	Reduction in Feet for every 100 Feet. (Approximative.)	
Degrees.		Ft.	In.
4	1 in 15	0	3
6	1 " 9½	0	6
7	1 " 8	0	9
8	1 " 7	1	0
10	1 " 6	1	6
11½	1 " 5½	2	0
14	1 " 4½	3	0
16¼	1 " 3¾	4	0
18¼	1 " 3	5	0
20	1 " 2¾	6	0

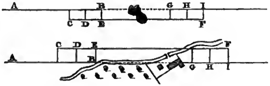
The necessary reduction, as estimated on the ground, is effected as the measurement proceeds, by putting the chain forward the exact number of feet denoted by the table as due to the angle, or the rate of inclination. This mechanical method possesses this advantage, that the crossing of the fences or natural boundaries, and the position of the offsets, is at once entered in the field-book with the required reduction. But we repeat, when perfect accuracy is sought, and when the survey is extensive, the angles of inclination should be observed, and the proper deduction obtained by computation, and allowed when the work is being plotted. This part of the subject is fully explained when treating of levelling and the interior filling-in of a trigonometrical survey.

Practical Methods of Measuring Inaccessible Distances, and of avoiding Obstacles in the running of Lines.

Cases of obstruction in the measurement of a line offered by the intervention of trees, buildings, rivers, lakes, &c., are readily overcome by practical geometry, even without the aid of an angular instrument. But when the difficulty cannot be surmounted simply by the chain, it is always better to make use of some angular instrument, which, with the aid of plane trigonometry, will enable the surveyor to solve all difficulties.

In ranging his lines, the surveyor should be careful to dispose them so that they shall, if possible, pass clear of trees, houses, and other impediments; if, in spite of all his efforts to the contrary, he finds it impracticable to avoid them altogether, he may proceed thus for passing them.

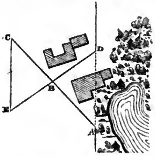
The measured line A B being obstructed by a tree, a brook, or a house, &c., staves are set up at C, D, and E, at equal distances from the measured line, and far enough from it to enable the new line C D E F, to pass clear of the obstacle. This new line, parallel to A B, is then measured till the obstruction is passed, when by setting other staves G, H, I, at distances from the second line equal to those first set out, a return is made to the direction of the original line, which is pursued as before.



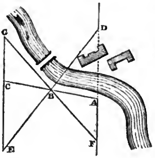
When the obstacle thus avoided is a tree, it is called a "sight tree," and for the purpose of facilitating future

reference, it is marked in a particular manner, by an arrow-head or otherwise, cut at or near the points where the direction of the line meets the tree both in front and rear.

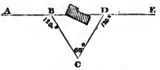
Another method of passing such obstacles is by the construction of equal triangles: thus, let A D be the direction of the line under measurement, the further progress of which is interrupted at A. From A measure A C in any direction, and leave a central mark B; from D, an accessible point on A D, measure a line D B E, making B E equal to B D, then C E will be equal to A D, the distance required. This method is inapplicable if the line has not been previously ranged and determined by signals fixed beyond D, and visible from it.



The following method, embodying the same property of equal triangles, may also be adopted, under different circumstances. At A and D erect staves, measure a line A C, at any angle with A D, and leave a central mark B. Measure a second line F B G, making B G equal to F B, join G C, and produce it to E, the point of intersection of the lines D B and G C; C E will be equal to A D, the distance required. In this case also, points ranged in the continuation of A D, must be visible from D.

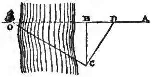


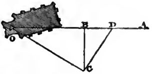
To avoid a similar obstacle, using an angular instrument. The measured line A B is interrupted at B; make



the angle A B C equal to two-thirds of two right angles, and proceed along B C far enough to clear the obstacle. At C, measure the angle B C D equal to one-third of two right angles, and proceed along C D to a point D, making C D equal to B C; at D, measure the angle C D E equal to two-thirds of two right angles. The triangle B C D is by construction equilateral, each angle being equal to one-third of two right angles; hence B D is equal to B C or C D; and D E is continued in the direction of the original line.

Required along the line A B produced, the distance B O, inaccessible to direct measurement with the chain. At B raise the perpendicular B C of any convenient length, by making with the chain a triangle whose sides are in the ratio of 3, 4, and 5, (see page 7). At C, range in the same manner the line C D perpendicular to C O, and produce it to meet A B in D; measure B D. Then because B C D and B C O are equiangular, (Euc. 4. VI.)



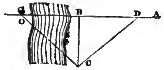


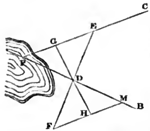
$$BD : BC :: BC : BO,$$

and (Euc. 16. VI.)

$$BO = \frac{BC^2}{BD};$$

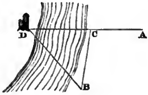
Or, using an angular instrument: at B raise B C perpendicular to A B, making B C of a convenient length; at C make the angle B C D equal to the angle B C O; measure B D, B D is equal to B O.



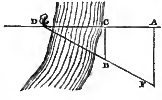


If two lines meet in a lake, river, or building, &c., the distances to their point of intersection are determined thus: Through any convenient point D, on the measured line B D, range E D F, making D F equal to E D; set out also a line G D H, making D H equal to D G; join F H, and produce it to cut B D in M, then D M is equal to D P, the distance required. On the line P E, the distance P G is also known, being equal to H M.

A ready method of determining a distance across a river is offered by that property of the triangle which consists in the external angle being equal to the two interior and opposite angles. Thus, on the line A D, the distance C D is required. At C measure any angle A C B, (it should not be less than 90°, for the sake of accuracy; see theorem with reference to well-conditioned triangles;) and setting the instrument to half that angle, proceed along the line C B, until the object D subtends with C the angle set; then C D B is an isosceles triangle, having the side C B equal to the side C D, the distance required.



C D may also be measured by raising at A and C the perpendiculars A F, C B. A F is measured of any convenient length, and C B is produced to meet the line F D in B. C B and C A are measured. Then by similar triangles, (Euc. 4, VI.)

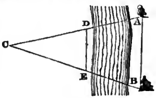


$$(AF - CB) : CB :: (AD - CD) : CD,$$

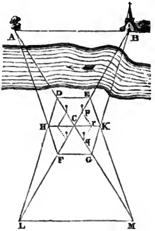
and (Euc. 16, VI.)

$$CD = \frac{CB \times CA}{AF - CB}.$$

To measure the inaccessible distance A B. Find by any of the methods above described the distances A C, B C, and take D C and E C in the same proportion; then D E is parallel to A B. Measure D E, and an application of ratios will give the distance A B. This method, which is exceedingly laborious, is given as a contrast to the following, remarkable for its simplicity.



Take any convenient point D, and, in the direction of A D produced, measure D G, leaving a central mark at C; and from C, in the direction B C, measure C F and C E each equal to C D. Set out H K parallel to D E or F G, as follows: place the centre of the chain at C, and fix the extremities at p and q in the lines C B, C G; stretch the chain from C in the direction C K, marking the point r (the middle of the chain), the line C r K is the parallel required. To get this parallel more correctly, repeat the same operation on the opposite side towards C H. Mark H and K, the points of intersection of H K with F A and



G B. Set out the lines D H, E K, and produce them to L and M, their intersections with B F and A G produced, then L M is equal to A B.

To find the height of an inaccessible tower built on an inclined plane, without the use of an angular instrument. Let A B (fig. 1) be the height required: from any accessible point E (fig. 2) measure the line E F, leaving a central mark at D. From F proceed in the direction F B to any accessible point G, from which measure G D H, making D H equal to D G: join E H, and produce it to C, where it meets B D produced, then C D is equal to D B. The length C B being thus obtained, at any point L erect a pole of such a length that the eye placed at C shall see the top of the pole P in the line C A, then measure C L and the height of the pole. By similar triangles,

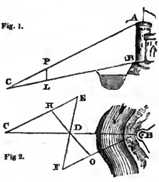


Fig. 1.

Fig 2.

$$C L : L P :: C B : B A,$$

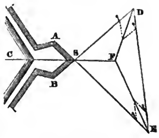
therefore,

$$B A = \frac{L P \times C B}{C L}.$$

Many other methods, more or less simple, may be contrived to solve these questions and others of a like nature,—they all resolve themselves into the principle of constructing on the accessible surface triangles either equal, or similar, to others resting on the inaccessible distances.

We shall conclude with the description of a method of determining in the field, without angular instruments, the direction of the capital of a bastion,—*i. e.*, a line dividing the work into two equal and similar parts. Let A S, B S,

be the faces of an inaccessible work; it is required to bisect the angle A S B.



Take any line D E, intersected in D and E by the prolongation of the faces B S, A S. Bisect the angles E D S and D E S by means of the chain, and produce the lines D F, E F, until they intersect in F; the line F S C bisects the angle D S E, and consequently the angle A S B. For the lines bisecting the angles of a triangle meet in a common point of intersection.

Plotting.

The scale to which a survey is to be plotted must first be determined: as, for instance, six inches to a mile, or three chains to an inch, or any other dimension suitable to the object of the survey.

The following are among the scales usually adopted for the special objects referred to:—

Two chains to an inch, or $\frac{1}{1584}$th of the actual size, is a scale suitable for plans of building-grounds or valuable property, on which it is required to measure minute portions of land.

Three chains to an inch, or $\frac{1}{2376}$th of the actual size, is well adapted to the plotting of surveys of parishes or estates. It is the scale adopted by the Tithe Commissioners for their first-class maps.

Scales of 100 feet and 200 feet to an inch are convenient for engineering purposes, owing to the facility they afford for decimal calculations.

For extensive surveys smaller scales are adopted. The Ordnance survey of Ireland is plotted and engraved in outline to a scale of six inches to a mile, $\frac{1}{10560}$th of the actual

size: the same scale is now being adopted for the remainder of the north of England, the southern part having been published on a scale of one inch to a mile, or $\frac{1}{63360}$th of the actual size.

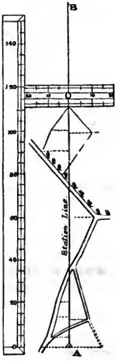
Plans and sections for projected lines of inland communications, to be deposited with the Houses of Parliament previous to the bringing in of the bills, are required by the standing orders to be drawn to scales, not less than 4 inches to the mile for the plan, and 100 feet to the inch for the section.

To assist in giving more precise ideas of the relative proportions of the scales used, and described in such various ways, it were to be desired that the proportion they bear to the actual size should be given in fractions, and thus expressed in a language as universal as numerical notation. The practice is general on the Continent.

To receive the plotting of the survey, strong drawing-paper is stretched on a board, or mounted on linen. The scale adopted for the plotting of the survey is then ruled on the plan, to serve for future reference, because the paper is liable to alter its dimensions through hygrometrical changes of the atmosphere after it has been cut off from the board. The experience obtained by the examination of the great number of maps inspected by Captain Dawson, of the Tithe Commission, shows generally a contraction in the length of the lines as first laid down; the average of which, on the scale of three chains to an inch, or $\frac{1}{2376}$th, is from one-fourth to one-half per cent., and requires therefore an allowance varying from one-half to one perch per acre.

The principal triangles are first laid down in pencil by the intersections of their sides. The lengths of these may be taken from a diagonal scale, and laid down with a pair of compasses; but if their lengths exceed the span of common compasses, they should be laid down by means of a beam-compass graduated to inches, and having a vernier which reads to one-hundredth of an inch. The points of intersection of the sides of the smaller triangles, and of all

proof-lines, are then pricked off, and marked in pencil with the corresponding letters taken from the rough diagram in the field-book. The detail is afterwards plotted, proceeding on the paper in the same order as that which was followed in the field. A long scale with bevelled edges, divided into inches and parts of inches as the scale used for the plan may require, is kept, by means of weights, at a fixed distance from, and parallel to, the main line,—the zero of the scale being opposite the starting point of the line. A shorter scale, divided on the edges in the same manner as the first, with its zero point in the middle, is made to slide along the fixed scale at right angles, while its central point keeps in coincidence with the station-line. The scales in general use for this purpose have been made of ivory; but, as they are very liable to irregular contraction and expansion, we would recommend well-seasoned box-wood scales. Engine-divided scales also, engraved on pasteboard, possess the recommendation of cheapness.



Mr. Drake, with the view to obviate the confusion and errors arising from the contraction of paper, manufactures for each map engine-divided scales on a portion of the mounted paper on which it is proposed to draw the map. The scale with which the plan has been plotted is kept with it, and as it has been made of the same material as that used for the map, it is affected in the same proportion by atmospheric changes.

The lengths entered in the

middle column of the field-book are measured on the first scale, and the perpendicular offsets on the offset scale: each point, thus determined by two ordinates, is marked on the plan with a fine-pointed pencil, or pricked with a fine needle. These points are from distance to distance joined by pencil-lines. Proceeding in this manner from side to side of each triangle, according to the order followed in the field-book, the whole of the survey is laid down in pencil.

The principal triangles are then ruled in with very faint red lines, to be preserved and exhibited as constituting the basis of the whole work. The various boundaries and objects are drawn in Indian ink with the steel drawing-pen: straight lines, rectangular boundaries, such as those of buildings, &c., are ruled; irregular boundaries, such as streams and winding fences, are drawn freely by the hand, the steel-pen being held upright, with its broad edge parallel to the direction to the line to be drawn.

Dwelling-houses are tinted with light flat tints of carmine; out-buildings with Indian ink; streams, rivers, lakes, &c., are tinted with light Prussian blue, laid in flat tints, a second tint being passed along the edges next to the upper boundary of the plan, to make them somewhat darker than the rest; trees are etched in Indian ink with the pen, or washed in with the brush; houses, roads, canals, bridges, &c., are drawn, as shown in the plate of characteristic signs.

Plans are usually drawn with their tops to the north. As we suppose the surveys to have been up to this time accomplished without angular instruments, we have given (see note, on setting out *meridian line,* Chapter III.) the means of setting out on the ground an approximate meridian line, by a simple mechanical process. The line thus set out in the field from the apex of one of the triangles can be connected by one or more tie-lines with one of the sides; and the survey may then be plotted with the north upwards.

Lastly, the writing required on the plan is to be disposed in parallel directions from east to west, with the exception of the names of rivers, canals, chains of mountains, &c., which are to be adapted to their natural sinuosities. The disposition of the writing requires ingenuity; and it is always to be borne in mind that the utility of a plan depends very much on that facility of reference which is obtained by the relative keeping of the names. The size of the letters must in some degree depend on the situations in which they are placed; but as a general rule, the names should be so written as to be legible at distances proportionate to the importance of the places. Thus, the names of hundreds should be legible at greater distances than those of parishes, villages than single houses, gentlemen's houses than cottages, &c.*.

On the Computing of Areas.

The superficial content of land is generally expressed in statute acres, roods, and perches. The acre is equal to 10 square chains of 66 feet in length, which chain, called Gunter's chain, is divided into 100 links: an acre, therefore, is equal to $(66)^2 \times 10 = 43{,}560$ square feet, or equal to 100,000 square links; the rood is one-fourth of an acre, and the perch one-fortieth of a rood. When the computation of acreage therefore is the object, the linear distances should be measured or expressed in links, the area thence obtained being in square links, is reduced to acres by a simple inspection, by cutting off with a decimal point the last five figures, the remaining figures to the left representing the acres; the decimal fraction, multiplied by 4, gives the roods; and the decimal part of this last product, multiplied by 40, gives the poles or perches. The table annexed gives the roods and perches, answering to the decimals, by simple inspection.

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* Colonel Colby's *Instructions for the Interior Survey of Ireland.*

A DECIMAL TABLE

For the use of LAND SURVEYORS, showing the Decimals answering to every Rood and Perch in the Acre.

P.	Dec.	1 R.	2 R.	3 R.	P.	Dec.	1 R.	2 R.	3 R.
1	,006	,256	,506	,756	21	,131	,381	,631	,881
2	,012	,262	,512	,762	22	,137	,387	,637	,887
3	,019	,269	,519	,769	23	,144	,394	,644	,894
4	,025	,275	,525	,775	24	,150	,400	,650	,900
5	,031	,281	,531	,781	25	,156	,406	,656	,906
6	,037	,287	,537	,787	26	,162	,412	,662	,912
7	,044	,294	,544	,794	27	,169	,419	,669	,919
8	,050	,300	,550	,800	28	,175	,425	,675	,925
9	,056	,306	,556	,806	29	,181	,431	,681	,931
10	,062	,312	,562	,812	30	,187	,437	,687	,937
11	,069	,319	,569	,819	31	,194	,444	,694	,944
12	,075	,325	,575	,825	32	,200	,450	,700	,950
13	,081	,331	,581	,831	33	,206	,456	,706	,956
14	,087	,337	,587	,837	34	,212	,462	,712	,962
15	,094	,344	,594	,844	35	,219	,469	,719	,969
16	,100	,350	,600	,850	36	,225	,475	,725	,975
17	,106	,356	,606	,856	37	,231	,481	,731	,981
18	,112	,362	,612	,862	38	,237	,487	,737	,987
19	,119	,369	,619	,869	39	,244	,494	,744	,994
20	,125	,375	,625	,875	40	,250	,500	,750	

The area of the principal triangles in the survey should in all cases be computed from the length of their sides (reduced to their horizontal bases), as obtained from the field-book. The operation (see investigation of theorem in note*) is simple in all cases, but rapidly performed with logarithmic tables. The contents of the fields and other

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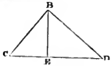
* Formulæ for Areas of Triangles.



Let A = area of triangle B C D

h = B E, the height of the triangle,

B C = d, B D = c, C D = b,

then $A = \frac{b \cdot h}{2}$, but $h = d$ sin. C,

therefore

$$A = \tfrac{1}{2}\, b \cdot d \text{ sin. C. } (1)$$

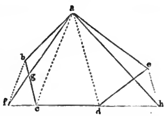
Again, sin. B : sin. D : : b : d, whence

$d = \frac{b \text{ sin. D}}{\text{sin. B}}$; by substitution, we have

inclosures could be similarly obtained by computation without measurement on the plan, but the process would be extremely laborious. Their area is instead obtained from measurement on the plan, and a comparison of the sum of the areas of each inclosure with the area of the whole as deduced by direct computation from the field-book, guards against the introduction of any casual errors of importance.

The area of the fields, or inclosures, might be calculated from the plans, by reducing the irregular polygons, constituting the inclosures, to triangles equivalent in area, as in the following example.

In the annexed irregular polygon *a b c d e*, draw a pencil line from *a* to *c*, and with a parallel ruler, draw the pencil line *b f* parallel to *a c*, cutting *c d* in *f*; join *a f*, then the area of the quadrilateral *a f d e* is equal to the area of the original figure. For, the triangles *a b f*, *c b f*, being on the same base and between the same parallels, are equal; from each take away the common triangle *b g f*, the remainders, *abg*, *cfg*, are equal. But in the alteration of the figure,



$$A = \tfrac{1}{2}\,\frac{b^2 \sin. C \sin. D}{\sin. B} = \frac{b^2}{2}\,\frac{\sin. C \sin. D}{\sin. (C+D)} \quad (2)$$

To obtain the area in terms of the sides; substituting in equation (1) the value of sin. C = 2 sin. ½ C cos. ½ C (trig.) we obtain

$$A = b\,d \sin. \tfrac{1}{2} C \cos. \tfrac{1}{2} C; \text{ but, making } s = \frac{d+b+c}{2}$$

$$\text{(trig.) } \sin. \tfrac{1}{2} C = \frac{\sqrt{(s-b)\ (s-d)}}{d\,b}, \text{ and}$$

$$\text{(trig.) } \cos. \tfrac{1}{2} C = \frac{\sqrt{s\,(s-c)}}{d\,b}; \text{ substituting, we have}$$

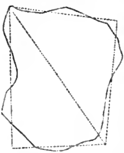
$$A = b\,d\,\frac{\sqrt{s\,(s-b)\ (s-c)\ (s-d)}}{b^2\,d^2},$$

$$A = \sqrt{s\,(s-b)\ (s-c)\ (s-d)} = \text{area of triangle.}$$

cfg has been substituted for *abg*, therefore the area of the quadrilateral *afde* is equal to the area of the original figure. In the same manner, by drawing *eh* parallel to *ad*, intersecting *cd* in *h*, and joining *ah*, the area of the triangle *afh* is made equal to the area of the quadrilateral *afde*, and consequently equal to the original figure, the contents of which are obtained by multiplying the base *fh* by half the height of the triangle.

It is evident that, whatever may be the number of sides of the polygon, a similar process will reduce it to an equivalent triangle; but it is no less manifest that the method would be very tedious if the boundary of the inclosure or figure were very crooked, presenting the additional objection of inaccuracy owing to the number of intersections to be obtained through the mechanical assistance of a parallel ruler.

A more rapid method, generally adopted hitherto, consisted in equalizing the boundaries by the eye, by applying the straight edge of a transparent piece of horn, or of gum paper, in such a manner that the small parts cut off from the crooked figure by it, should be equal to those taken in. Pencil lines were then ruled along the edge of the horn, and the figure reduced to triangles the contents of which were easily computed.

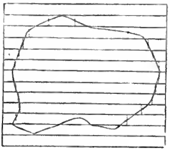


Various other means of obtaining contents from the direct measurement of plans have been adopted; we shall limit our description to the following method by which the necessity for calculation is avoided; and which from its extreme simplicity has now generally superseded all others. Its first general application was made in the Tithe Commission office, about two years ago.

A scale represented in the annexed diagram is made

equal in length to 50 chains as plotted on the plan: it is divided by transverse lines into 20 parts; each part would therefore, between parallels 1 chain in width, represent a space equal to 1 rood, 2 of the parts, between the same parallels, would represent a space equal to 2 roods, and so on for the whole length of the scale, which itself would, between the same parallels, represent a space equal to 20 roods or 5 acres. A slider is attached to the scale admitting of motion in the direction of its length; it bears a vernier scale with spaces equal in length to 1 part on the primary scale (or to $2\frac{1}{2}$ chains) divided into 40 parts, each of which spaces would, therefore, with a height of 1 chain, represent $\frac{1}{40}$ of a rood, or 1 perch. The slider carries in a projecting frame a transparent piece of horn with a fine line drawn across its centre at right angles to the direction of the length of the scale, and having also 2 fine lines, parallel to the length of the scale, ruled 1 chain apart. To apply this scale to the measurement of areas, a strong transparent paper (called horn paper) with parallel lines ruled 1 chain apart is laid over the field whose area is required, and kept fixed by a weight. The cross line in the projecting frame, being placed opposite the zero of the scale, is brought to that part of the left-hand boundary of the fence which is inclosed between the upper parallel lines of the transparent paper; and the scale is laid in a direction parallel to these lines by the coincidence of the two parallels on the sliding frame. If the cross line do not coincide with the boundary, it is so placed by moving the scale as to exclude on the left as large a space as it includes on the right; the scale is then kept fixed by the pressure of the hand, while the slider is removed to the right-hand boun-

dary of the field, and the cross line adjusted in a similar manner as before: so far, the number of divisions on the scale passed over by the cross line, in its movement from left to right, indicates the number of roods and parts of roods included in the field between the upper parallel lines. The *slider* remaining fixed in position, the *scale* is moved downwards 1 chain, that is, to the next lower division of the parallel lines, and the cross line is brought, as before, by the movement of the scale, to coincide with the left-hand boundary; and the coincidence or compensation being established, the sliding frame is moved



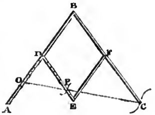
to bring the cross line into coincidence with the right-hand boundary; the division opposite the cross line then indicates the number of acres, roods, and perches comprised within the two upper parallel divisions. When the slider has, by a repetition of the process, passed over the whole length of the scale, 5 acres have been measured. The operation is then continued by moving the slider from *right* to *left* (the equalization commencing on the right-hand side) until the slider having reached the starting point or zero of the scale, 10 acres have been measured, the quantity measured being indicated by the scale itself. The mechanical operation is continued and repeated in a similar manner until the whole field has been measured.

On Copying Plans.

It rarely happens that one copy only of a plan is required. When the lines and boundaries are regular, duplicates may be made by laying the plan upon the sheet of paper or vellum on which the copy is to be drawn, and by pricking with a fine needle, through all the angular points necessary to define the figures: the punctures being then connected by pencil lines, the plan is finished by tracing these in ink.

This method is not suitable to the transfer of irregular or curved boundaries; the most accurate and rapid way of copying these is by means of the instrument called a copying glass. It consists of a large piece of plate glass, set in a frame of wood, which can be inclined to any angle, when the lower side rests on a table. On this the original plan and the fair sheet of paper are laid, and the frame being raised to a suitable angle, a strong light is thrown by means of tin reflectors or otherwise on the under side of the glass, whereby every line in the original plan is seen distinctly. The copy is at once made in ink, and finished while being traced.

When reduced plans are required, they are first drawn in pencil by means of the pentagraph, to any proportion wanted. The instrument consists of a jointed rhombus B D E F, made of brass, and having the two sides B D, B F extended to double their length; the side D E and the branch D A are marked from D with successive divisions, D O being made to B O always in the ratio of D P to B C. Small sliding boxes for holding a pencil or tracing point



are placed at P and C, and secured in their positions by screws; the point O is made the centre of motion, and rests on a fulcrum or support of lead; and the tracer is generally fixed at C, while the pencil is lodged in P. From the property of similar triangles, the three points O, P, and C must range in the same straight line, which is divided at P in the ratio required. While the point C, therefore, is carried along the boundaries of any figure, the intermediate point P will trace out a similar figure, reduced in the proportion of O C to O P or of O B to O D, the proportion required*.

By changing the relative positions of the tracer and pencil, the figure would be enlarged in the same proportion; but errors are so much increased by the enlarging of plans, that it ought not to be resorted to except in cases where the field-books are either lost, or do not afford the data required to plot on a larger scale, and when there are no means of repeating the survey.

* LESLIE's *Geometry*, page 431.

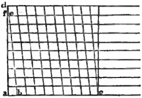
NOTE. Through the kindness of Captain Dawson of the Tithe Commission Office, we are enabled to give the accompanying plate of conventional signs, which are admirably adapted for detailed plans.

CHAPTER II.

SURVEYING INSTRUMENTS.

When a line is to be divided into equal parts, so numerous and minute, that they would be indistinct, some method, not embodying direct subdivision, must be adopted in order to estimate those minute fractional parts. The diagonal scale, an ingenious application of the property of similar triangles, described in Euc. 4, VI., was among the earliest methods resorted to for this purpose.

In the annexed figure, the line $a\,c$ is divided into any number of equal parts, or into n times $a\,b$, and a space equal to each of these parts is subdivided into n secondary parts, by means of diagonal lines, of any arbitrary length, raised from the points marking the primary divisions. These diagonals decline, in their entire lengths, from the perpendicular



by intervals equal to one of the primary parts, and they are cut transversely into n equal parts by equidistant lines parallel to $a\,c$. In the triangles $a\,d\,b, f\,d\,e$, we have

$$b\,d : a\,b :: d\,e : e\,f,$$

but $d\,e$ is by construction equal to the n^{th} part of $b\,d$, therefore $e\,f$ is equal the n^{th} part of $a\,b$, or equal to $\frac{a\,c}{n^2}$.

This subdivision by diagonals is still in general use*, and constituted the first improvement in dividing astronomical instruments. In this part of its application it has been superseded by the vernier scale, the simplest and most ingenious of the methods hitherto invented for the minute

* Leslie's *Geometry*.

subdivision of lines. It obtains this object by measuring the *differences* between the divisions of two approximating scales; one of which is fixed, and called the primary scale; the other moveable, and called the vernier.

If a space, on the primary scale, be divided into a given number of parts, equal to $n - 1$, and a space, equal in length to the first, be divided on the moveable scale into a number of parts equal to n, these parts will each be smaller than the first by the n^{th} part of a division on the primary scale.

For, let $a =$ the length of a division on the primary scale,

$b =$ the length of a division on the movable scale;

then by hypothesis,

$$(n - 1)\,a = n\,b, \text{ or}$$
$$n\,a - a = n\,b, \text{ and}$$
$$a - \frac{a}{n} = b;$$

that is, b, a division on the movable scale, is smaller than a, a division on the fixed scale, by the n^{th} part of a.

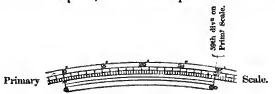
The edges of the two scales being applied to each other, so that the extreme end of the vernier, which is marked $\overset{\circ}{\downarrow}$ and is called the index, coincides with a division on the fixed scale, then the quantity of aberration at the first division of the vernier will be $\frac{a}{n}$, at the second $\frac{2\,a}{n}$, and so on to the n^{th} division, when the aberration becomes equal to $\frac{n\,a}{n}$ or a; and therefore a coincidence always obtains simultaneously at the first and last division of the vernier. In moving the vernier forward, the quantity of aberration at each successive division will diminish by the extent of the space moved over, until a new coincidence takes place at the first division, which coincidence shows the amount of displacement to have been $\frac{a}{n}$. In the same manner, the amount of displacement may be made equal successively to $\frac{2\,a}{n}$, $\frac{3\,a}{n}$ ..... $\frac{(n-1)\,a}{n}$, and, finally, $\frac{n\,a}{n}$ or a, when the

space moved over becomes equal to one of the divisions on the primary scale.

If 1 inch, for example, be divided on the primary scale into 10 equal parts, and a space on the vernier equal to 9 of these parts be itself divided into 10 equal parts, the difference between a division on the primary scale and a division on the vernier will be equal to $\frac{1}{10}$ of the first, and therefore equal to $\frac{1}{100}$ of an inch.

A general rule for reading with a vernier may be expressed thus: observe the number of parts on the primary scale that is equal in length to the same number *increased by one* on the vernier; this last number is tho denominator of a fraction whose numerator is unity, and which expresses the subdivision of the *parts* on the primary scale.

For example, on the common 5-inch theodolite, the circle forming the primary scale is divided into 360 degrees, each degree being subdivided by shorter lines into 20 minutes. Now a space, equal in length to 59 of these subdivisions, is adopted for the length of the vernier, and divided into 60 parts; each of these parts on the vernier is,



Vernier, divided into 60 parts.

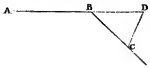
therefore, smaller than those on the circle by $\frac{1}{60}$, or by 20 seconds. In reading with the instrument, an account is first taken of the degrees and fractional parts of a degree, as shown by the index of the vernier; then the eye is passed along the vernier until some one of its lines, coinciding with any line on the primary scale, is found. The quantity to be added to the first approximate measure is then obtained by counting the number of divisions included between the zero or index of the vernier and the coincident

line, each division corresponding, as before explained, to 20 seconds. To simplify the reading, every third line on the vernier is made longer than the rest, and represents minutes, while the shorter intervening lines represent one-third of a minute or 20 seconds.

THE THEODOLITE.

The method of surveying by the chain alone is applicable only to surveys of comparatively small extent, and simple in their outlines; for, even in small surveys, the intervention of towns, villages, high inclosures, or other obstacles may be found to render the measurement of right lines by the chain extremely difficult; and, by isolating different portions of the work, to cause inaccuracies that may be avoided by the use of an angular instrument.

Angles, it is true, may be determined by the chain alone, by measuring the sides of small triangles disposed for the purpose, thus: let A B represent a line measured to a station B, from whence a second line B C, forming an angle with A B, is to be measured. To determine the angle A B C, prolong A B to D, make B C equal to B D, (in order to construct a well-conditioned triangle,) and measure the chord D C: the three sides of the triangle B D C being known, the angle D B C or its supplement A B C is determined. This is a method which ought, however, rarely to be resorted to; for, no time is gained by its adoption, and the chances of error are considerably multiplied, owing to the numerous additional lines to be measured. Moreover, it is to be observed, that angles can in general be measured in the field more correctly with an instrument than the

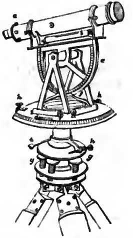


length of lines with the chain, especially over uneven ground or in an inclosed country.

The instrument in general use, for the purpose of measuring angles in surveying, is the theodolite, of which there are several constructions, differing slightly in the arrangement and adjustments of the parts. A detailed and clear description of these varieties, as also of nearly all the instruments used in surveying, is given by Mr. Simms, in his valuable *Treatise on Mathematical Instruments*. The following is a description of the theodolite in most general use.

The theodolite consists of two circular plates, the upper turning freely on the lower, and both having a horizontal

a Diaphragm with adjusting screws.
b Spirit-level.
c Lower circular plate.
d Vernier to lower circular plate.
e Vertical arc.
f Vernier to vertical arc.
g Parallel-plate screws.
h Clamp and tangent-screws.



motion by means of a vertical axis. This axis, with the view to render the motion of the circular plates independent one of the other, is made of two parts, external and internal; the former secured to the lower plate, and the latter to the upper plate. The circumference of the lower plate is divided into 360 degrees, and parts of a degree, as described when treating of the vernier; and at the extremities of a diameter of the upper plate are fixed two verniers *.

* Simms, *on Mathematical Instruments*.

These circular plates are intended to measure horizontal angles, *i.e.*, angles in a plane parallel to the horizon,—they must therefore be adjusted in the horizontal plane. This adjustment is effected by means of four screws (called parallel-plate screws), set in pairs opposite to each other. Two spirit-levels, placed at right angles to each other, on the upper plate, serve to guide the moving of the screws.

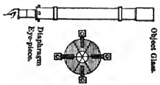
A spirit-level is a glass tube nearly filled with a liquid, generally spirit of wine, and hermetically sealed. The tube has a slight and regular curvature, (it being impossible to manufacture a tube mathematically straight*;) it is placed with the convex side upwards, and the bubble in that position occupies the higher central part. Several divisions marked

on the tube at equal distances on each side of the bubble, serve to indicate very slight deviations from horizontality; that level being most sensible which most nearly approaches to a mathematically straight surface.

The upper and lower horizontal plates are retained in any required position by clamp-screws; tangent-screws affording the means of fixing them with more precision than can be attained by the hand alone. A frame, resting on the upper plate, supports the axis of the telescope in angular recesses called Y's, from their resemblance to that letter. A horizontal motion, therefore, given to the telescope for the purpose of observing an object, may, by the above arrangement of the vertical axis and clamps, be communicated to one or both of the circular plates.

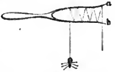
In the focus of the eye-piece and object-glass, and at right angles to the length of the telescope, are placed three lines formed of fine wires, or spider's web; one horizontal, the others crossing its middle point, so as to form an acute angle with each other. These wires serve to point the telescope with certainty to any object or part of an object.

* SIR J. F. W. HERSCHELL'S *Astronomy*.



Front view of Diaphragm.

When these wires are broken, an accident to which their fineness renders them liable, they are easily replaced by cobwebs in the following manner:—" A piece of wire is bent into a shape something like a fork, the opening *a b* being rather larger than the diameter of the diaphragm. A cobweb being selected, from the extremity of which a spider is suspended, it is wound round the fork in the manner represented in the sketch, the weight of the insect keeping it constantly tight. The web is thus stretched ready for use; and when it is required to fix on a new thread, it is merely necessary to put a little gum or varnish on the diaphragm, and adjust one of the threads to its proper position, as indicated by faint notches on the metal *."



From the lower part of the telescope is suspended a spirit-level, which, being more sensible than the levels fixed on the vernier plate, is used in the final adjustment of the circular plates to the horizontal position, previous to the taking of observations.

To the under part of the telescope is attached a vertical semicircular arc for the purpose of observing altitudes and depressions. The axis of this vertical arc rests, at equal heights above the vernier plate, on two points in the frames supported by it: consequently, when the upper plate is horizontal, the semicircular arc attached to the telescope is in a vertical plane. The angle of inclination of the tele-

* Lieut. Frome's *Trigonometrical Surveying*, p. 21.

scope is indicated by a fixed index and vernier attached to the upper plate, in the locus of a perpendicular let fall from the centre of the axis of the vertical arc. The vertical arc is adjusted and retained at any required angle of inclination by means of a clamp and tangent-screw. One side of the arc is graduated into degrees and parts of a degree; the other side shows the difference between a hypotenuse of 100 units and the base in right-angled triangles, calculated to degrees of inclination of the hypotenuse from 0° to 45°. By means of this table of differences, the required deduction can be accurately made when actually measuring over an inclined plane, in order to reduce it to its horizontal base.

The above constitute the essential characteristics of the instrument, but a compass-box is usually attached to the upper circular plate. It is used sometimes for noting the bearings with the meridian of different stations, as a check on the measured angles. Also, in setting out a long straight line, the extremities of which are invisible from intermediate points, the bearing of the line may with advantage be taken at its extremities and at intermediate points, in order to serve as a check, but as a check only, on the straightness of the line.

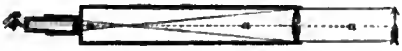
The instrument is fixed by means of a screw on the staff-head of the three legs which form its stand. Beneath the centre of the staff-head, a hook is attached for the purpose of suspending a plummet to guide the observer in placing the instrument exactly over the station at which the observations are to be made.

Adjustments of the Theodolite.

The first point to be attended to in observing with the theodolite is, to draw out the tube of the eye-piece till the cross wires appear clearly defined, and afterwards to correct any optical displacement of the cross wires arising from a lateral change in the position of the eye. This displace-

ment is called instrumental parallax, a general term used in science to denote the difference between the true and apparent places of an object. If the image of a distinct object remain fixed when the eye is moved laterally out of the optical axis, no parallax exists. If, on the contrary, the image of the object does not remain fixed, draw out the tube of the eye-piece more or less until the required stability of the image takes place. As an additional precaution against error from parallax, it is well always to observe as nearly as possible through the middle of the aperture of the eye-glass.

The next adjustment is that of the line of collimation, *i. e.*, a line (*a a* in diagram) passing through the point



of intersection of the cross wires fixed in the focus of the object and eye-glasses, and the centres of those glasses. The adjustment consists in making this line of collimation to coincide with the axis of the cylindrical rings on which the telescope turns. This is necessary, because the observations are made and registered under the supposition that such a coincidence exists, as it is only by reference to the cylindrical rings or external tube of the telescope that we estimate the direction of the optical axis, whether in azimuth or altitude. This adjustment is effected as follows:—

Make the intersection of the cross wires to coincide with some well-defined part of a distant object; then turn the telescope half round in its Y's, till the level lies above it; if the same point be not again covered by the centre of the wires, move this centre one-half of the amount of deviation by means of the diaphragm screws, and correct the other half by elevating or depressing the telescope. If the coincidence of the wires and the object then remain perfect in both positions of the telescope, the line of collimation in altitude or depression is correct; but if not, the operation must be repeated until the adjustment is satisfactory. A

similar process will adjust the line of collimation in the vertical plane.

The next adjustment is that which fixes the level attached to the telescope in a position parallel to the rectified line of collimation, or longitudinal axis of the telescope. The necessity for this adjustment is evident, as the only means of judging of the horizontal position, or of the amount of deviation from it, of the optical axis, is by reference to the spirit-level, which is assumed as being parallel to the said axis. To effect this adjustment, the clips that retain the telescope in its place being open, and the vertical arc clamped at or near zero, bring the air-bubble of the level to the centre of the tube by turning the tangent-screw, which moves the vertical arc; then, reverse the telescope in its Y's, end for end. If the bubble do not return to the middle of the tube, bring it back one half by the capstan-headed screw placed at one end of the tube (to elevate or depress that end of the level), and the other half by the tangent-screw that acts on the vertical arc. This process is repeated until the adjustment is perfect.

Another adjustment is that whereby the circular plates are placed horizontally. This is also necessary, because any deviation from the horizontal plane would introduce a proportional error in the measurement of what are assumed as being horizontal angles. Place the instrument as nearly level as can be done by the eye, fasten the lower horizontal plate by its clamp, leaving the upper plate free, and move the latter so as to place the telescope over two of the parallel-plate screws; then bring the bubble of the level under the telescope to the middle of the tube by the tangent-screw of the vertical arc; next turn the upper plate 180° from its former position; if the bubble do not return to the middle, half the difference is to be corrected by the parallel-plate screws, and half by elevating or depressing the telescope by means of the tangent-screw. The same operation is repeated over the other pair of parallel-plate screws, so

that the air-bubble of the spirit-level attached to the telescope shall remain constantly in the centre of the tube, in whatever position it is turned. The two small levels on the vernier plate are then to be adjusted, by means of the capstan-headed screws that keep them in place.

The vernier of the vertical arc is next to be attended to: it is correct if it point to zero when the foregoing adjustments are perfect, and any deviation in it is easily rectified by the attached screws; or if the deviation be small, note the quantity of deviation as an index error, and apply it, + or −, to each vertical angle observed. This deviation is best determined by repeating the observation of an altitude or depression in the reversed positions, both of the telescope and vernier-plate: the two readings will have equal and opposite errors, and the half of their difference will be the index error.

On Measuring Angles with the Theodolite.

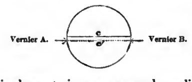
The adjustments before described having been carefully examined and rectified, the theodolite is placed exactly over the station from whence the angles are to be taken, by means of the plumb-line suspended from its centre. It is then set level by the parallel-plate screws, bringing the telescope, with the vertical arc clamped at zero, over each pair alternately. Clamp the lower horizontal limb in any position, and direct the telescope to one of the objects to be observed, moving it till the object and cross wires coincide; then clamp the upper limb, and by its tangent-screw make the intersection of the wires exactly bisect the object; now read off the two verniers, which are respectively marked A and B, noting the degrees, minutes, and seconds of A, and the minutes and seconds of B, and take the mean of the two readings.

Next, release the upper plate, and move it round until the

telescope is directed to the second object, whose angular distance from the first is required, and by the clamp and tangent-screws make the cross wires to bisect the object. Again read off the verniers, and the difference between their mean, and the mean of the first reading, will be the angle required.

	Vernier A.	Vernier B.	Mean.
1st reading	142° 36′ 20″	36′ 40″	142° 36′ 30″
2nd reading	228° 43′ 20″	43′ 0″	228° 43′ 10″
		Difference	85° 6′ 40″

The object of reading off the two verniers placed diametrically opposite to each other, is to counteract the effects of eccentricity in the two circular plates and their axes, the principle of the instrument requiring that the circles should be concentric with the axes on which they are made to turn, and with each other. Now whatever be the extent of this deviation, its effect is neutralized on the result of observations depending on the graduation of the limb, by taking the mean of the two readings as above described; for the effect of eccentricity is always to increase one such reading by exactly the same quantity by which it diminishes the other. However, in cases where great accuracy is required, the observer should not rest satisfied with one measurement of the angle, even thus corrected from the error due to eccentricity: for the result is liable to two other sources of error,—that of graduation and that of observation itself.



To whatsoever degree of perfection the construction of astronomical or geodesic instruments has been brought, it constitutes only an approximation to geometrical accuracy; and among the varied operations of this high branch of

mechanical art, none presents greater difficulties than the accurate division of the circumference of a circle turned in metal into 360 equal parts, and these again into smaller subdivisions. "The attainment of perfect accuracy in this work has hitherto baffled the utmost stretch of human skill and industry; nor, if executed, could it endure. The ever-varying fluctuations of heat and cold have a tendency to produce, not merely temporary and transient, but permanent, uncompensated changes of form in all considerable masses of those metals which alone are applicable to such uses; and their own weight, however symmetrically formed, must always be unequally sustained, since it is impossible to apply the sustaining power to every part separately: even could this be done, at all events force must be used to move and fix them, which can never be done without producing temporary, and risking permanent change of form*."

The errors of the second class, called errors of observation, arise from inexpertness, defective vision, atmospheric indistinctness, momentary instrumental derangement due to the want of a firm basis to support the legs, slips in clamping, looseness of screws, &c.

To obviate, in a great degree, these errors, the "principle of repetition," an invention of Borda, is applied. To repeat an angle, therefore, after making the second bisection, as above described, leave the upper plate clamped to the lower, and release the clamp of the latter; now move the horizontal limb with the telescope to point to the first object, till the cross wires are in coincidence with it. Fixing it thus, release the upper plate, and turn the telescope towards the second object, and again bisect it by means of the clamp and tangent-screw of tho upper plate. "Let this process be repeated as often as is deemed advisable (suppose ten times); then will the final arc read off on the circular

* Sir J. F. W. Herschell's *Astronomy*, p. 66.

plate be ten times the required angle, affected by the joint errors of all the ten observations, but only by the same constant error of graduation, which depends on the initial and final readings alone. Now, the errors of observation, when numerous, tend to balance and destroy one another; so that, if sufficiently multiplied, their influence will disappear from the result. There remains, then, only the constant error of graduation, which comes to be divided in the final result by the number of observations, and is, therefore, diminished in its influence to one-tenth of its possible amount, or to less if need be. The abstract beauty and advantage of this principle seem to be counterbalanced in practice by some unknown cause, which, probably, must be sought for in imperfect clamping," and the straining of the parts consequent on the action of the tangent-screws.

The proof of accuracy of a number of horizontal angles, if they surround the station, is to add them all together, and their sum, if correct, will be 360°. Also, if they be taken at several stations (near enough to each other to make the spherical excess inappreciable), the sum of all the interior angles of the polygon formed by joining the stations by straight lines will be equal to twice as many right angles as the polygon has sides, wanting 4 right angles (Euc. Cor. 32, I.) Thus, if the figure have 3 sides, the sum of the interior angles will be equal to 180°; if 4 sides, the sum will be equal to 360°.

To measure angles of elevation or depression, unclamp the vertical arc, and direct the intersection of the cross wires of the telescope to the object. Note the reading of the vertical arc, and repeat the operation with the telescope turned half round in its Y's; that is, with the level uppermost; the mean of the two readings will neutralise the effect of any error that may exist in the line of collimation. When very great accuracy is aimed at, or when it is desired to ascertain the index error, two more readings may be

taken in a similar manner with the upper or vernier plate and telescope reversed longitudinally. If there be any index error, it will be equal to half the difference between the mean of both readings, which, if no error existed, would be equal. This method of obtaining the angles of inclination, by four readings, is free from the effects of any error that might exist in the adjustment of the line of collimation, as well as from the index error of the vertical arc.

The magnetic bearing of an object is taken by simply reading the angle pointed out by the compass needle, when the object is bisected. With the common compass attached to the theodolite, (usually from 4 to 5 inches in diameter,) the angle cannot be obtained with certainty nearer than one degree to the truth.

In conclusion, it may be observed that the telescope generally used (see diagram, page 45), shows the objects inverted; the reason for the inversion is, that fewer glasses being required, objects are seen more clearly. Practice quickly renders the inversion of the image immaterial to the observer. However, a second eye-piece is usually provided, which shows objects in their natural position, and may be substituted at the eye-end of the telescope.

The Spirit-Level.

The knowledge of the principal parts of the theodolite and its adjustments, will enable the student to understand by a simple inspection the use and adjustments of the Spirit-Level.

The instrument consists of a telescope precisely similar to that of the theodolite, and requiring analogous correction for parallax and adjustment of the line of collimation. To the telescope is attached a spirit-level, the longitudinal axis

of which should be likewise, and for the same reason as in the theodolite, parallel to the line of collimation. Finally, the object of the instrument being to obtain a constantly horizontal axis of vision, the telescope or line of collimation is brought into a horizontal plane by means of the parallel-plate screws.

Parallax. The correction for parallax is made by bringing the tube carrying the eye-glass into such a position that the point of intersection of a distant object shall not be altered by a slight vertical or lateral movement of the eye.

Collimation. In the construction of the instrument known as the Y spirit-level, the telescope rests on supports similar to those which retain the telescope of the theodolite:—for this construction, the line of collimation is rectified, as in the theodolite, by turning the telescope half round in its Y's after the first observation has been made, and noting whether the horizontal cross wire intersects the

same point. The cross wires in the diaphragm of the level are arranged as shown in the diagram; the horizontal wire marks the intersection of the horizontal visual ray with the staff; the two vertical wires serve to direct the telescope so that the staff shall be seen between them, and thus be in the axis of the lenses, and likewise in a vertical direction.

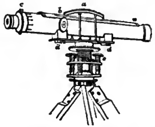
In the Y level, the spirit-level is suspended beneath the telescope, as in the theodolite, and requires a similar adjustment to bring its longitudinal axis parallel to the line of collimation, which is performed by reversing the telescope end for end in its Y's, and altering the screws as described in the adjustments for the theodolite.

But in those modifications of the instrument known as "Troughton's Improved Level," and "Gravatt's Level," and which, from superior compactness, greater stability of the adjustments, and increased optical power, are more

generally used, the adjustments of the line of collimation and of the spirit-level are made in a different manner.

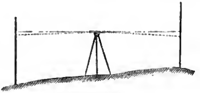
By a reference to the annexed diagram, which represents "Troughton's Improved Level," it will be seen that the telescope is attached to the bearing-frame, and the spirit-level firmly fixed to the tube of the telescope. This construction requires a different method of making the adjustment.

a Compass-box.
b Spirit-level.
c Diaphragm with adjusting screws.
d Screws to adjust spirit level.
e Parallel-plate screws.



ment. In the first place, the spirit-level is adjusted perpendicularly to the vertical axis of the instrument by correcting half of the observed deviation by the screws *d*, and the other half by the parallel-plate screws, until the telescope can be moved round in any direction without any material change taking place in the position of the bubble.

The spirit-bubble being thus adjusted, the instrument will fulfil the object of giving a line in any direction parallel to the plane of the horizon, provided that the line of collimation be itself parallel to the longitudinal axis of the spirit-level. This is examined and corrected as follows:—



Let two staves be held upright at a distance of 400 or 500 feet from each other, and the instrument set up exactly

midway between them. The line of collimation (whether it be correct or not, as appears from the figure,) will intersect the two staves on the same level; the difference of reading being noted, the instrument is removed to a point near one of the staves, and if in this new position the difference of reading on the staves be the same as before, the line of collimation is correct; if not, the error is corrected by raising or depressing the diaphragm, (as the case may be,) until the difference of reading on the staves shall be, in this second position of the instrument, the same as it was in the first.

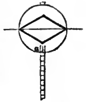
The application and mode of action of the parallel-plate screws need no explanation.

Levelling Staves.

The levelling staff simply consists of a rectangular rod or rods divided into feet and hundredths of a foot, by black lines appearing on a white ground. The hundredth part of a foot can be distinctly read at a distance of 400 to 500 feet with "Gravatt's Level." Two such staves are required with each instrument.

In certain operations of levelling, (as explained under that head,) it is required to determine the intersection of the horizontal axis of vision on the staff at much greater distances. For this purpose, a sliding vane with a rhombus, marked by thick black lines on a white ground, is attached to the staff, on which it forms a conspicuous

object. The angular points of the rhombus can be intersected with great precision by the horizontal wire, at a considerable distance. By means of signs, the staff-holder is directed to raise or lower the vane until the horizontal wire bisects the rhombus; and the reading, as shown by the index *a*, is noted and registered



by the assistant. When this index is placed, as in the figure (which is the usual construction), below the centre of the rhombus, the divisions, instead of commencing with zero at the bottom of the staff, are graduated at a distance above zero, equal to the distance from the index to the centre of the rhombus.

CHAPTER III.

TRIGONOMETRICAL SURVEYING.

The characteristic difference between a trigonometrical survey and a survey as described in our first chapter, consists in this:—that, whereas in the latter, the relative positions of the principal and secondary stations are ascertained by direct *linear measurement;* in the former they are ascertained by trigonometrical *calculations* based upon the direct linear measurement of a single line only, (called therefore a base line,) combined with the observation of angles as hereafter described. This trigonometrical operation is indispensable to obtain accuracy in the performance of any extensive survey. The first stop consists in the measurement of the base line.

Measurement of Base Line.

The measurement of a base line, from which the sides of the triangles of an extensive series are to be calculated, is a most difficult operation*, and one in which every refinement which mechanical ingenuity can devise has been of late adopted, with a view to obtain almost mathematical accuracy. The length of the base is made to depend in general on the proposed length of the sides of the triangles which are to be deduced from it; but circumstances seldom allow it to exceed from seven to eight miles in extent, as its position has to be selected on an even plain, as nearly as possible horizontal, and otherwise conveniently adapted for purposes of measurement.

* Sir J. F. W. Herschell's *Astronomy;* Trigonometrical Survey of England and Wales; Delambre, *Base du Système Métrique.*

Standard Measures.

A necessary precaution at the outset of the operation consists in carefully comparing with a well-known standard the particular instrument or unit to be employed as the medium of measurement. "By the word standard, or by a standard yard or standard metre, is meant a certain extent of space in one direction, in the abstract, without any reference to wooden rods or metallic bars. But as it is impossible to measure a line without some material standard, we are compelled to adopt as the practical definition of a yard, metre, &c., the length of a certain bar (usually of metal) called a standard. But as from change of temperature, the length of the bar (as compared with the length of others not subjected to the same trial) is found to change, then we must specify the degree of temperature under which this certain bar must be placed in order to present the exact length required. The length of a base, measured with a standard at a higher temperature than that specified, must be increased; and, if measured at a lower temperature, must be diminished, according to the ratio of increase or diminution in the length of the bar ascertained by experiment as due to one or more degrees of temperature*."

The temperature to which English standards are referred is 62° Fahrenheit. In the measurement of the arc to serve as the basis of the metre, the French geometricians adopted the temperature of 13° Réaumur, or 61¼° Fahrenheit.

There have been in England, to within a late period, several standard measures of authority, differing all slightly one from the other;—but at the recommendation of the Commissioners of Weights and Measures appointed in 1818, the parliamentary standard, prepared by Bird in 1760, was adopted as the foundation of all legal weights and measures, and declared the "unit or only standard measure of exten-

* AIRY's *Figure of the Earth.*

sion of the United Kingdom." It had originally been the intention of the Commissioners to recommend for adoption the standard used for the trigonometrical survey of Great Britain; but a careful comparison of the length of this with other standards of authority having shown that it differed considerably from the latter, the first intention was laid aside.

The following shows the result of the comparison made by Captain Kater of the respective lengths of six standards that had been frequently referred to up to that period. Assuming the length of Colonel Lambton's standard, used on the Indian survey, at 36 inches, the lengths of the others were found as follows:—

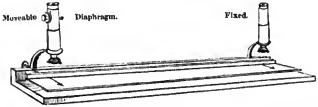
	Inches.
Colonel Lambton's standard . . .	36·000000
Bird's standard of 1760	36·000659
Sir George Shuckburgh's scale . .	36·000642
Ramsden's bar (used as the standard for the trigonometrical survey) .	36·003147
General Roy's scale	36·001537
Royal Society's standard	36·002007

The determination of the Commissioners to adopt Bird's standard presented this advantage, that, differing from Sir G. Shuckburgh's by a quantity so small that the two could be considered identical, and the length of the metre having been converted into English measure by comparison with Sir G. Shuckburgh's scale, the expression for the metrical measures of the Continent converted into parts of Sir George Shuckburgh's scale, may be assumed as equally correct in giving their value with reference to Bird's or the Parliamentary standard*.

A standard measure having been thus agreed upon, it was necessary to prepare others from it, to be used for the purpose of sizing all measures employed in commerce or

* Philosophical Transactions for 1821.

scientific researches. Captain Kater, in a Memoir read before the Royal Society in 1825, gives a description of his mode of adjusting the duplicate standard yards,—an operation of considerable difficulty, as it involves the necessity of transferring, without error, a given distance from one scale to another. This difficulty was overcome by the following ingenious contrivance. A disk of gold, with a minute dot in its centre, was let into the surface of a brass bar near one of its extremities: at the distance of thirty-six inches from this dot a hole was drilled through the bar, to receive an accurately fitting, but moveable, brass plug made slightly conical, and projecting on the under surface of the bar. Into the upper surface of the brass plug a gold disk was inserted, and a second dot was made in this disk as nearly as possible at the distance of 36 inches from the first, but so that it should not be in the centre of the moveable disk. The micrometrical apparatus for measuring minute distances having been adjusted of the proper length, the



intersection of the cross wires of one of the micrometers was placed over the fixed dot, the moveable dot was then brought under the intersection of the cross wires of the second micrometer, by means of a lever inserted into the projecting part of the moveable brass plug, in order to move it round its axis. A drill passed through a hole made in the side of the scale into the brass plug, served to keep the dot in its place.

Captain Kater subsequently found that an alteration made in the straightness of the standard, by the insertion of

a card under its middle part, or of a card at each of its extremities, made a very sensible difference in the distance between the dots, (no less than ·0016 of an inch for the greatest difference,) the distance being increased when a card was placed under the middle of the bar, and diminished when cards were placed under the extreme ends. To remedy this serious inconvenience was indispensable; for no means could ensure that the standard, when referred to, should always be placed upon the same level table, and on the same part of it. Reasoning from the natural supposition that a plane, intermediate between the upper and lower surfaces, suffered neither expansion nor contraction by a change of level of the parts, the thickness of the bar was reduced one-half at the extremities in which the gold disks were inserted. This alteration solved the difficulty, for it was found that pieces of card, placed in any position underneath the bar thus altered, made no appreciable change in its length. Four standards were thus prepared of the same length as the imperial standard yard, and one was deposited at the Exchequer, Westminster; one at Guildhall, London; one in Edinburgh; and one in Dublin. But these, being perfect copies, were to be referred to only upon extraordinary occasions for scientific purposes; four other standards were therefore made at the same time, with great precision, (although not with the extreme care bestowed on those first described,) for the purpose of being deposited in the same places, to be used as the standard yards, by reference to which those employed in commerce were to be sized or adjusted.

The length of the pendulum, vibrating seconds in London, had been estimated at 39·13929 inches expressed in parts of the imperial standard yard; and as this was supposed to be both a correct and an unalterable quantity, it was prescribed in the Act that the length of the standard yard should be restored by reference to the length of such a pendulum, should accident at any time lead to the destruction of the legal standard deposited in the House of Commons.

This legal standard having been shortly afterwards, in the burning of the Houses of Parliament*, so far injured that it was impossible to ascertain from it, with the most moderate accuracy, the statutable length of 1 yard, the attention of the legislature has again been directed to the formation of a new standard. The commissioners appointed to report on the subject have come to the conclusion that, many elements in the reduction of the pendulum experiments having been ascertained to be doubtful and erroneous, the course prescribed by the Act would not reproduce the length of the original standard. They therefore recommend that in this, as in future exigencies of a like nature, the legal standard be restored by reference to material copies which have been, or shall be, carefully compared with the parliamentary standard, and not by the results of experiments referring to natural constants. They propose that the Royal Astronomical Society's scale, the Royal Society's scale, and the three-feet bars belonging to the Board of Ordnance, and now in the custody of Colonel Colby, be in the present case used as the material bases from which the new legal standard is to be formed, being convinced that by an advantageous combination of their values, the original standard can be restored without sensible error.

Instruments used in measuring Base Lines.

Different instruments have been employed at different times as the means of measuring base lines. Deal rods, which had been originally used in England and on the Continent, were soon laid aside in exact operations, as experience demonstrated that they were liable to sudden and irregular changes from dryness, humidity or other

* Report of the Commissioners for the Restoration of the Standards of Weight and Measure, 1841.

causes*. General Roy found that deal rods, not varnished, were lengthened about half an inch in 300 feet by exposure for one night to moisture. However, for ordinary surveying operations, deal rods saturated with boiling oil, and afterwards covered with a thick coat of varnish, will be found sufficiently exact, as this process tends to protect them in a great measure from the effects of hygrometric changes in the atmosphere †. When deal rods are employed, their ends should be protected by metal caps which prevent their wearing, and ensure a more perfect contact.

In the measurement of a base on Hounslow Heath in 1784, glass rods, which expand or contract less than steel, iron or brass, were substituted instead of deal. Their extremities were furnished with caps of bell-metal, connected with the rods by springs; the caps were brought in each operation to a certain mark on the rods, in order that unequal compression of the rods (tending of course more or less to affect their length by bending them) might be avoided. The change was crowned with success, as was proved by a subsequent measurement of a distance of 1000 feet with the same glass rods on the one hand, and with a steel chain of perfect workmanship on the other, from the result of which it appeared that the difference would have been little more than half an inch upon the whole base of 27404 feet, had it been measured with each respectively. The same experiments further showed that hollow glass tubes were less liable to sudden expansion and contraction than solid glass rods.

As the above test had also proved the steel chain to be as accurate as the glass rods, and as it was more convenient to use, it was subsequently employed in the measurement

* Trigonometrical Survey of England and Wales.

† PUISSANT'S *Géodesie*.

of bases of verification in different parts of the kingdom. In using the steel chain, a drawing post and a weight post are required, a given weight being always applied to one end of the chain while the other end is fixed to the drawing post. The chain is made to rest in deal coffers, in order to obtain a perfectly level surface: thermometers are laid at different distances along the chain, and the coincidence at the end of the chain made only when all the thermometers read alike. These thermometers have to be used, because the chain is affected in the direction of its length by changes of temperature; and certain reductions, hereafter explained, have to be made for the purpose of obtaining the true length of the base for a fixed point of the thermometer.

The French geometricians, who conducted the operations for the determining of the metre*, employed rods of platina and brass, invented by Borda, which embodied an important improvement, inasmuch as the use of thermometers was superseded, by making the measuring rods themselves act as metallic thermometers. It was ascertained by experiment, that, with every degree of the centigrade thermometer†, the expansion of platina amounted to 0·000008565, and that of brass to 0·000017843 of unity. A rod of platina 12·78 feet in length was overlaid by a rod of brass about 6 inches shorter, and both were rivetted together at one extremity only. The other extremity being

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* Delambre, *Base du Système Métrique*; Puissant's *Géodesie*.

† The centigrade thermometer has the space between the temperature of melting ice and boiling water divided into 100°: — to convert the degrees of this thermometer into those of Fahrenheit, we have, centigrade $\frac{9}{5} + 32 =$ Fahrenheit.

Réaumur's thermometer has the space between the temperature of melting ice and boiling water divided into 80°:—to convert the degrees of Réaumur into those of Fahrenheit we have, Réaumur $\frac{9}{4} + 32 =$ Fahrenheit.

left free, the effect caused by a change of temperature was at each moment made manifest by the difference of expansion of the two metals. The measure of this difference

was obtained by means of the scale S attached to the platina bar, and sliding inside a groove made at the end of the brass bar. Each division of the scale S was equal to $\frac{1}{20000}$ of the length of the brass rod (or $\frac{1}{136}$ of an inch nearly). By means of the vernier attached to the brass rod, each division was subdivided $\frac{1}{10}$th and the observer was thus enabled to read by means of powerful microscopes the $\frac{1}{2000000}$ of the length of the bar (or $\frac{1}{1360}$ of an inch nearly). The difference of expansion between the brass and platina being for one degree 0·000017843 − 0·000008565 = 0·000009278, a quantity nearly double of 0·000005, or $\frac{1}{200000}$, which quantity the vernier could indicate, the change due to an alteration in the temperature, of a quantity smaller than one degree, could be detected and noted. The bases measured in this operation were referred to the temperature of 13° Réaumur, or 61¼ Fahrenheit.

In the measurement of the Irish base of between 7 and 8 miles on the plain of Magilligan, "and in which the greatest possible error is supposed not to exceed 2 inches," a beautiful and novel apparatus was devised by Col. Colby in which, by using compensating expansions, he obtained an unalterable linear measure, and therefore obviated the necessity of noting the temperature as well as all subsequent reductions. "Two bars, one of iron, the other of brass, 10 feet long, were placed parallel to each other and rivetted at their centres, it having been previously ascertained by numerous experiments that they expanded or contracted, in their transitions from heat to cold and the reverse, in the

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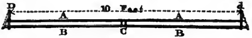
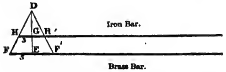
* FROME'S *Trigonometrical Surveying*, page 3.

proportion of 3 to 5. The brass bar was coated with some non-conducting substance, to equalize the susceptibility of the two metals to change of temperature. Across each extremity of these combined bars was fixed a tongue of iron, with a minute dot of platina, almost invisible to the naked eye, and so situated on this tongue that under every change of contraction or expansion the dots at each extremity always remained at the constant distance of 10 feet*.



"Let A be the iron bar, the expansion of which is represented by 3, B the brass bar the expansion of which is 5, the two being rivetted together at their centre C; D and *d* are two iron tongues pinned on the bars so as to admit of their expansion, with the platina dots at D and *d*. The tongues are, by construction, made perpendicular to the rods at a mean temperature of 60° Fahrenheit, and the expansion taking place from their common centre, when A expands any quantity which may be expressed by 3, B expands at the same time a quantity equal to 5, and the inclination of the tongues is changed, the dots D and *d* remaining unalterably fixed at the exact distance of 10 feet."

The distance between the bars being given, the position of the dots D and *d* is found by means of the following proportion.



$$DG : GH :: DE - DG : FE - HG.$$

* Frome's *Trigonometrical Surveying*, p. 3.

Assuming the distance G E equal to 4 inches we shall have D G : 3 :: 4 : 2,

$$D\,G = \frac{12}{2} = 6 \text{ inches.}$$

Mathematically speaking, the point D would only remain unalterably fixed, under the supposition that the tongue should alter in length in the ratio of the distances D F, D F′; for, supposing the bars, by contraction, to bring the tongue into the position F′H′D, the distances F′H′ and H′D are increased, whereas the reduction in the temperature causes the corresponding portions of the tongue to be diminished. Were the effect appreciable in practice, it could be in a great measure avoided by making originally the tongues perpendicular to the bars, at a temperature lower than that to which they would be subjected during the measurement. But, as a proof that the effect is altogether inappreciable in practice, even after a great number of repetitions, it will be found that, supposing a change of temperature of 20° from the point at which the tongues have been set perpendicular to the bars to take place, the expansion or contraction in the brass bar (that is the distances F E, or F′E) would be little more than $\frac{1}{100}$th of an inch. If we suppose, with our data, the 20° to be an increase above 60°, the increase of the $\frac{1}{100}$th of an inch constituting the base of the triangle D E F would cause in the hypotenuse an increase less than that which the iron tongue would undergo through the expansion due to the assumed change of temperature.

It is evident from the construction of these, as well as that of Borda's bars, that the dots or divisions at the extremities of the bars could not, if desired, be brought either into actual contact or coincidence. The distance between Borda's bars was measured by means of a microscopical apparatus, somewhat resembling the instrument represented in the diagram, page 59. The focus of each micrometer was adjusted and fixed over the dot or division marking the

extremity of each bar, and the distance between the foci of the microscopes ascertained*. In Colonel Colby's apparatus, the micrometers were separated by a constant distance, being attached to the unalterable points of short compound bars, (similar to the larger compensating bars above described,) each of which was so laid as to have its dot in the focus of the corresponding micrometer†.

The unit of measure having then been selected according to the degree of care to be bestowed on the work, the base line should be ranged with a theodolite or transit instrument, and traced by means of pickets driven into the ground, at convenient intervals, in the same vertical plane.

The necessity for ranging straight lines with great accuracy frequently occurs in practice. For this purpose, the theodolite or transit instrument having been fixed very firmly, the axis of the vertical arc or of the pivots of the telescope must be adjusted to a truly horizontal position with great care. Marks are then fixed in the ground, at different distances, in a continuous vertical plane, as far as the power of the telescope will permit; the instrument is then taken forward to within three or four marks or pickets of the extremity of the line ranged, and fixed correctly over one of them by means of the plummet, and by the intersection of the cross wires of the telescope directed to the back and forward pickets successively. Boards 12 or 15 inches square, with concentric black and white rings painted on both sides of the board, make good ranging marks. The mark is made to move in horizontal grooves cut in two posts driven firmly into the ground; and when the centre of the mark has been brought into the line, it is fastened by wedges to the posts. A picket is next driven into the ground, its position being determined by the plummet, and a notch cut in it under the centre of the mark to secure the line. By this method, Messrs. Dixon and Mason mea-

* PUISSANT'S *Géodesie*.

† FROME'S *Trig. Surv.*

sured with accuracy a continuous straight line of 100 miles in length in the provinces of Maryland and Pennsylvania*. When the base line has been accurately ranged, it is measured; and, as a necessary precaution in common operations, it is measured at least twice, in opposite directions. When the base has been measured in a direction inclined to the horizon, a section of the line is carefully taken with the spirit-level or theodolite, as hereafter explained, for the purpose of reducing the line to its horizontal base.

Reduction of a Base to its Value at a given Temperature.

If a brass bar or unit of measure be compared at the temperature of 70° Fahrenheit, and found equal in length to a platina bar at the same temperature, but which bar is, when at the temperature of 62°, equal to the standard yard; and if a base equal in length to A be measured with the brass bar at the mean temperature of 75°, the number of yards contained in the base A will be obtained as follows:—

Let the amount of expansion undergone by a metal bar of a given length, in passing from a given temperature to a given higher temperature, expressed in terms of the length of the bar at the lower of these two temperatures, be called the *relative expansion*.

Let $P = 0{\cdot}00000476$ (Borda), represent the relative expansion of platina; and

$B = 0{\cdot}00001049$ (Roy), represent the relative expansion of brass for each degree of Fahrenheit; then, because the platina bar is equal to the standard yard at the temperature of 62°, it will, at the temperature of 70°, exceed the standard yard by 8 times P. Hence, in order to

* *Phil. Trans.*, 1768. Memoir by Messrs. Dixon and Mason.

ascertain the temperature at which the brass bar will itself be equal to the standard yard, making the length of the bars at the temperature at which they were compared, equal to *one*, we shall have

$$\text{Yard in platina} = 1 - 8\ \mathrm{P}, \text{ and}$$
$$\text{Yard in brass} = 1 - x\ \mathrm{B},$$

x representing the number of degrees to be deducted from 70° to obtain the length of a yard in the brass bar. As brass contracts and expands more rapidly than platina, the temperature at which the brass bar would be equal to a yard will of course be higher than 62°;—the exact point will be found by the following equation, derived from the above:

$$1 - 8\ \mathrm{P} = 1 - x\ \mathrm{B}; \text{ or}$$
$$x = \frac{8\ \mathrm{P}}{\mathrm{B}} = \frac{8\ (0{\cdot}00000476)}{0{\cdot}00001049} = 3{\cdot}63^\circ.$$

Hence the temperature at which the brass bar would be equal to the yard is $70^\circ - 3{\cdot}63^\circ = 66{\cdot}37^\circ$ Fahrenheit.

But the base A having been measured at a mean temperature of 75°, that is, when the bar exceeded a yard by $(75^\circ - 66{\cdot}37^\circ) \times \mathrm{B} = (8{\cdot}63^\circ) \times \mathrm{B}$, the length of the base is short by a quantity equal to A times $(8{\cdot}63^\circ) \times \mathrm{B}$, and its true length is equal to

$$\mathrm{A} + \mathrm{A} \times (8{\cdot}63^\circ) \times \mathrm{B}.$$

Assuming A = 15,000 times the length of the brass bar, and taking the value of B as given above, the true length of the base will be

$$15{,}000 + 15{,}000\ (8{\cdot}63^\circ \times 0{\cdot}00001049), \text{ or}$$
$$15{,}001{\cdot}35793 \text{ yards.}$$

In this example, therefore, the correction amounts to upwards of 4 feet, a quantity much too great to be neglected, when a base is to be measured with extreme care.

Example 1.—Let us take, as an example of this reduction, the observations connected with the measurement of the base on Hounslow Heath in 1791 *.

This base was measured with two chains of blistered steel of 100 feet in length, which were found, by the result of nine experiments, to expand 0·0075 inch for 1 degree of Fahrenheit.

The base contained 274 chains and 1·755 feet; hence its apparent length was

27401·755 feet.

At a temperature of $51\frac{1}{2}°$, the chain A was found to exceed 100 feet by

0·114 inch,

and the chain B by

0·058 inch.

Now the temperature of the standard, (a bar of cast-iron,) from which these lengths were ascertained at the temperature of $51\frac{1}{2}°$, having been 54° when its own length was determined; and the expansion due to 100 feet of cast-iron, being for 1° Fahrenheit 0·0074 inch, whereas that of the steel chain was found to be 0·0075 inch; their difference is

0·00010 inch for 1°, and
0·00025 inch for $2\frac{1}{2}°$.

To obtain, therefore, the lengths of the chains at the temperature of 54°, we have:

	Feet.	Inches.	Inches.	Feet.	Inches.
Chain A at 54° =	100	+ 0·114	+ ·00025 =	100	+ 0·11425,
Chain B at 54° =	100	+ 0·058	+ ·00025 =	100	+ 0·05825.

Out of the 274 chains contained in the base, 236 were measured with the chain A, and 38 with the chain B; we have therefore the correction for the excess of the chains' lengths above 100 feet,

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* *Trig. Survey*, vol. i., p. 21.

		Inch.	Inches.
Correction for chain A	= + 236	(0·11425)	= 26·9630
" " B	= + 38	(0·05825)	= 2·2135
		Sum	= 29·1765

And dividing by 12 to reduce it into feet, we obtain

+ 2·4314 feet

as the quantity to be added to the apparent length.

Again, the mean reading of the thermometers during the measurement was 70·65°, that is 16·65° above the temperature at which the standard was determined; each chain was therefore too long at the time of the measurement by a quantity equal to

16·65 times 0·0075 inch,

the amount of expansion due to 1°; that is, by a quantity equal to 0·124875 inch;

and this multiplied by 274, the number of chains contained in the base, will give the amount to be added, in order to obtain the value of the base at the temperature of 54°. Combining these results, we have

	Feet.
Apparent length . . . . . =	27401·7550
Correction for excess of chains' length above 100 feet, at temp. of 54° . .	+ 2·4314
Correction for mean temp. above 54°	+ 2·8513
True length of base at 54° . .	27407·0377
But in Great Britain it is customary to refer all measures of length to a standard temperature of 62°; to effect this, we multiply 0·0075 inch by 8, multiplied by 274, which gives the amount to be deducted from the length at 54°; that is, $\frac{8 \times 274 \times (0{\cdot}0075 \text{ inch})}{12} =$	− 1·3700
From which we obtain for the value of the base at 62°	27405·6677

Example 2. To reduce the base measured on Salisbury Plain, in 1794, to the temperature of 62°, from the following data.

	Feet.
The base contained 366 chains, wanting 9·939 feet	36590·0610
The base was measured with the same chain A as before; taking its excess above 100 feet at the temperature of 54° as before, the correction for excess will be	+ 3·4837
Five thermometers were applied to each chain, and the sum of all the degrees shown by the thermometers was 146,051°; hence we have for the mean reading of the thermometers 79·8°, and therefore the correction for mean temperature is, $\frac{25 \cdot 8^\circ\ (0 \cdot 0075 \text{ inch}) \times 365 \cdot 9}{12} =$	+ 5·9001
Again, to reduce the base to the standard temperature of 62°, we have $\frac{8\ (0 \cdot 0075 \text{ inch}) \times 365 \cdot 9}{12} =$	− 1·8278
Hence value of base reduced to 62° =	36597·6170

Note. This base was measured on an inclined surface, and a further reduction of its length to the horizontal base must therefore be made, as shown in page 76.

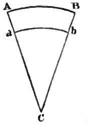
Example 3. To reduce the base measured on Misterton Carr, Yorkshire, to its value at the temperature of 62°.

	Feet.
Apparent length of base, 259 chains of 100 feet, + 8 chains of 50 feet, + 38·3 feet	26338·30
The chain of 100 feet was known to be 0·1255 parts of an inch too long at a temperature of 54°; therefore $\left(\frac{0{\cdot}1255 \text{ inch}}{12}\right)$ 259 to be added .	+ 2·71
The 50 feet chain was found to be 0·0471 parts of an inch too long at the same temperature; therefore $\left(\frac{0{\cdot}0471 \text{ inch}}{12}\right)$ 8 to be added . .	+ 0·03
Again, five thermometers having been applied at each measurement of the 100 feet chain, the sum of the degrees shown by the thermometers was 98083°, which gives for the mean reading $\frac{98083}{5} \div$ 263·383 chains = 74·5°; 54° being deducted from the mean reading, there remain 20·5° to be multiplied by 0·0075 inch and by 263·383 chains; to be added	+ 3·37
Finally, for the reduction to the temperature of 62°, we have $\frac{8\ (0{\cdot}0075 \text{ inch})\ 263{\cdot}383}{12}$ to be deducted	− 1·91
Length of base at temperature of 62° .	26342·50

Reduction of a Base to its Value at the Level of the Sea.

In order to compare together and connect bases measured at different elevations in distant parts of the country, it is necessary that they be referred to a common elevation. For this common standard of elevation, the level of the sea, in maritime states, naturally presents itself as the most suitable, admitting, by its very nature and universal access, of easy reference.

A base measured on an elevated plain is thus reduced to its proper measure at the level of the sea.



Let $R =$ radius of the earth, (supposed to be spherical,) at the level of the sea;

$R + h =$ radius at the level of the measured base;

$A =$ the measured base $A\,B$; and

$a =$ the reduced base $a\,b$.

Then, because similar arcs are in the same ratio as their radii*, we have

$R + h : R :: A : a$, and

$$a = \frac{R \,.\, A}{R + h},$$

and the difference between the measured and reduced base,

$$A - a = A - \frac{R \,.\, A}{R + h};$$

reducing both terms to a common denominator,

$$A - a = \frac{A\,R + A\,h - A\,R}{R + h}, \text{ whence}$$

$$A - a = \frac{A\,h}{R + h} = \text{difference required.}$$

* Leslie's *Geometry*, VI. 30.

As the level of the sea constantly changes by the effect of the tides, a conventional elevation must be agreed upon, to which all measurements performed in the same work shall be referred. The elevation adopted on the Ordnance Survey for the standard is that of low-water mark;—the most suitable for several considerations, but especially because the soundings on charts, (with which all extensive land surveys, in maritime states, sooner or later are connected,) must be referred, as hereafter explained, to the level of low-water.

Example 1. Assuming the mean elevation of the base at Hounslow Heath above the sea at 102 feet, and the measured base corrected as above to the temperature of $62° = 27405{\cdot}6677$ feet. Taking R, the radius of the earth, at 7,002,667 yards, required the measure of the base at the level of the sea.

$$\begin{aligned} A &= 27405{\cdot}6677 \text{ feet}, \\ h &= 102 \quad " \\ R &= 21{,}008{,}001 \quad " \end{aligned}$$

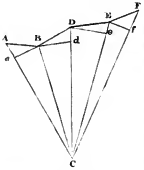
$$A - a = \frac{A\,h}{R + h} = \frac{(27405{\cdot}6677) \times 102}{21{,}008{,}001} = 0{\cdot}133 \text{ feet};$$

hence reduced base $= 27405{\cdot}6677 - 0{\cdot}133 = 27405{\cdot}5347$ feet.

Example 2. The base measured on Salisbury Plain, and corrected to the temperature of 62°, is as previously obtained $= 36597{\cdot}617$ feet; the elevation of Beacon Hill, one of its extremities, is 690 feet above the sea; assuming this for the present $= h$, and taking $R =$ the same quantity as above, required the measure of the base at the level of the sea.—*Answer*, 36596·4 feet.

Reduction of Bases to their Horizontal Values.

In the theorem in the previous page, the base A B, which it is required to reduce to the level of the sea, has been assumed as horizontal or concentric with the surface of the sea. But measured bases are usually more or less inclined to the horizon, and they must be reduced to their horizontal value before they are reduced to the level of the sea.



In the adjoining figure, let C be the centre of the earth, and A B D E F the measured base, of which different portions are at different elevations, and variously inclined to the horizon. The sum of all the corrections for the reduction of the lines A B, B D, D E, &c., to the horizon, will be the sum of the differences between those hypotenuses and the horizontal lines a B, B d, D e, &c. If the differences of elevation, as that between A and B for instance, have been obtained by the spirit-level, the height A a is given, and we have

$$a\,B = \sqrt{A\,B^2 - A\,a^2} \quad (1)$$

in which A B is the measured hypotenuse.

If the difference of level have been obtained by taking with the theodolite, or any angular instrument, the angle of elevation a B A, then

$$a\,B = \frac{\cos.\,B \times A\,B}{R} \quad (2)$$

On the Salisbury base, which we have taken as an example, the inclinations were measured with an angular instrument; its reduction to the horizon was therefore performed by

means of the second formula. The calculations may be disposed according to the following table.

Hypotenuses.		Angles of elevation or depression.			Log. cos. angles of elevation or depression.	Log. hypotenusal distances.	Log. horizontal distances.	Horizontal distances.
No.	Feet.	°	′	″				
1	100	7	52	30	9·9958849	2·0000000	1·9958849	99·057
2	100	11	31	40	9·9911670	2·0000000	1·9911670	97·986
3	100	10	5	0	9·9932396	2·0000000	1·9932396	98·456
4	100	7	25	20	9·9963468	2·0000000	1·9963468	99·162
5	100	5	41	50	9·9978494	2·0000000	1·9978494	99·506
6	700	4	49	30	9·9984583	2·8450980	2·8435563	697·519
7	600	4	18	40	9·9987694	2·7781513	2·7769207	598·302
8	300	3	48	30	9·9990399	2·4771213	2·4761612	299·338
9	300	3	13	0	9·9993152	2·4771213	2·4764365	299·527
10	100	0	9	0	9·9999985	2·0000000	1·9999985	100·000
11	100	2	27	30	9·9996001	2·0000000	1·9996001	99·908
12	100	0	58	30	9·9999372	2·0000000	1·9999372	99·986
13	300	0	5	0	9·9999995	2·4771213	2·4771208	300·000

A part only of the calculation is given to serve as an example: the total reduction amounted to 20·916 feet*, to be deducted from the length as before obtained.

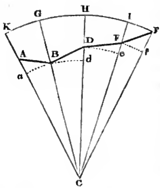
$$\begin{array}{r} 36596{\cdot}4 \text{ feet} \\ -\quad 20{\cdot}9 \;\;\text{,,} \\ \hline \end{array}$$

$$\text{Difference} = 36575{\cdot}5 \text{ feet.}$$

It may be observed with reference to this base, that the angles of inclination were determined with positive certainty to within 1 minute of the truth; and an error of 1 minute in these inclinations, supposing them all to lie one way, would produce only an error of 3 inches in the length of the whole base.

In the preceding investigation, the hypotenuses A B, B D, D E, &c., were all reduced to their immediate bases, but these bases are all at different elevations; they should therefore, in mathematical strictness, have been reduced to their respective values, as forming parts of an arc passing through a certain point in the base. Let the arc pass

* *Trig. Survey*, vol. i., p. 71.



through one of its extremities, F; draw the lines C A, C B, C D, &c., and produce them until they intersect the arc F K in K, G, H, &c., then will the lines K G, G H, H I, &c., be those to which the lines *a* B, B *d*, D *e*, &c., are to be reduced. This reduction is obtained from the ratio of the radii B C, D C, &c., to the constant radius F C. This last correction may, however, be generally omitted from its great minuteness: in the Salisbury base, which is nearly 7 miles in length, and in which one extremity is elevated 416 feet above the other extremity, the correction due to this consideration amounts only to half a foot.

Reduction to a Straight Line of a Base, the parts of which are in different Vertical Planes.

Bases, as previously observed, should, if possible, be chosen on open level plains, where they may be measured in continuous straight lines; but circumstances may occur to render it advisable to measure a base of verification, of which different parts shall be in different vertical planes. For instance, Captain Mudge and Mr. Dalby, in the progress of the Trigonometrical Survey, wished to measure a base line in South Wales*, on the level between Passage House and Cardigan; and being unable to find any spot, 4 miles in length, sufficiently unobstructed, they would readily have dispensed with the common practice of having the base one continued line, had they been able to find any point from which two right lines might have been measured, forming such an angle with each other, as to give, for the

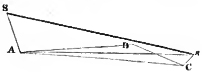
* *Trig. Survey*, vol. ii., p. 10.

third side of the triangle, a line of more than 5 miles in length.

On the continent, bases of verification have sometimes been measured in non-continuous straight lines, many of the great roads offering, from their character, advantages for the measurement of bases as to economy of time and expenditure, although they may not happen to be in the same straight line for a length sufficient for the required base.

The extremities of the base, when so disposed, may, moreover, be unsuited for stations of observation; in such a case, the stations are chosen as near the base as practicable, and their distances from the extremities measured with as much care as the principal base, of which they, in fact, form a part.

Let A B C be the measured base, and S *s* the stations situated near its extremities; the angles A B C, B A S or C A S, and A C *s* or B C *s*, are measured with extreme care.



In the triangle A B C, we have two sides and the contained angle; hence we obtain A C, and the angles at A and C.

In the triangle A C *s*, we have two sides and the contained angle; hence we obtain A *s*.

In the triangle A S *s*, we have two sides and the contained angle; hence we obtain S *s*, the base required.

Example. In a base of verification, measured by Delambre, on the road from Lieursaint to Melun*, we have

A B = 3945 French toises† = 25226·3 Eng. feet.
B C = 2131 „ = 13626·7 „
Angle A B C = 179° 11′.

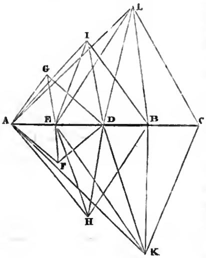
* PUISSANT's *Géodesie*, vol. i., p. 213.
† The French toise is equal to 6·3945 English feet.

Also, assuming B A S = 87° 25′
B C *s* = 92 30
A S = 1200 feet,
and C *s* = 360 „

required the length of the line joining the stations S *s*.—*Ans.* 38829 feet. (See p. 84, on the *condition* of this triangle.)

To Verify and Prolong a Base by Triangulation.

" Besides the marks at the extremities of a base line, which, if the base is to form the ground work of a survey of considerable extent, should be constructed so as to be permanent, as well as minute, intermediate points should be carefully determined and marked during the progress of the measurement, by driving strong pickets or making some clearly-defined mark. These marks serve for testing the accuracy of the different portions, and reciprocally comparing them with each other, thus:—let A B represent the portion of the base actually measured, and B C that to be added by calculation, for the purpose of extending the base to C, in order to obtain a more eligible termination. The points E, D have been marked during the measurement. The stations F G are selected, so that the angles at E may be nearly right angles, and the points themselves nearly equi-



distant from the line, and about equal to A E. Similar conditions determine the positions of H, I, K, and L. At A, as well as at every point previously marked on the base and selected on each side of it, angles are observed to every other point. With these data G E and E F are determined; from each of these E D is obtained by calculation; and from A E and E D, and A D as bases, I D and D H are obtained; and lastly, by similar processes, B L and B K are found as the mean results of many operations, all tending to check each other. B C is finally obtained from B L and B K independently, used as bases in the triangles B L C, B K C*."

In this manner, the Irish base, on the plain of Magilligan was prolonged about two miles, the termination of the north end of the base being ill-adapted to serve as a station for general observations of the angles.

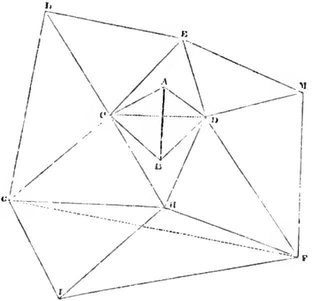
Triangulation.

A base having been thus measured with every precaution demanded by the nature of the work of which it forms a part, the next step is the triangulation, or the division of the country to be surveyed into a series of great triangles, the angles of which are placed at stations, clearly visible from each other. The angles are generally measured by means of the theodolite.

The following figure represents such a series of triangles. A B is the base, C, D, &c., are stations visible from both its extremities, and E, F, G, H, &c., other stations, on commanding points in the country, by the connexion of which the whole surface may be divided, as it were, into a net-work of triangles. Now in the triangle A B C, the angles A, B, C being observed, and one of the sides A B measured, the other two sides may be calculated by the rules of trigono-

* Frome's *Trigonometrical Surveying*, page 9.

metry; and thus each of the sides A C, B C, becomes in its turn a base capable of being employed as the known side of other triangles. All the stations may in this manner be



accurately determined and laid down, and as this process may be carried on to any extent, a map of the whole country may be thus constructed, and filled to any degree of detail required. The triangles ought not, however, to be laid down until their accuracy has been tested by the actual measurement of one or more of the distant sides, which are therefore called bases of verification. Care should be had during the progress of the work to calculate many of the sides of the triangles from several independent data, in order to prove the identity of the results*.

In this process it is necessary to be careful in the selec-

* Sir J. F. W. Herschell's *Astronomy*, chap. iii.

tion of the stations, so as to form triangles free from any *very* great inequality in their angles. For instance, the triangle H G F in the last figure, would be an improper one to determine the situation of F from observations made at H and G, because the angle F being very acute, a small error in the angle G or H would produce a great one in the position of F upon the line B F*. By referring to page 79, it will be seen that the base chosen by Delambre, on the road from Lieursaint to Melun, has the angle contained between

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* *Theorem.* To determine the relation of the parts in *well-conditioned triangles.*

Let A, B, C be the angles, and a, b, c the sides of a triangle, we have

$$a \sin. B = b \sin. A.$$

Supposing the side b to be the measured base, the value of which has been obtained with perfect accuracy, and the errors of observation to occur in the other terms of this equation; in other words, taking b constant, the other terms variable, and differentiating, we have

$$a\, d \sin. B + \sin. B\, d\, a = b\, d \sin. A,$$

but the differential of the sine of an arc is equal to the differential of the arc multiplied by the cosine of the arc; hence the equation becomes

$$a \cos. B\, d\, B + \sin. B\, d\, a = b \cos. A\, d\, A;$$

$$d\, a = \frac{b \cos. A\, d\, A}{\sin. B} - \frac{a \cos. B\, d\, B}{\sin. B}$$

but
$$\frac{b}{\sin. B} = \frac{a}{\sin. A}$$

Substituting, we obtain

$$d\, a = \frac{a \cos. A\, d\, A}{\sin. A} - \frac{a \cos. B\, d\, B}{\sin. B}$$

$$= a \cot. A\, d\, A - a \cot. B\, d\, B.$$

In this equation d A and d B may be supposed to represent the errors made in the measure of the angles A and B, whether they be errors of construction or observation.

If we suppose the errors both equal, and to be both either in excess or defect, we have for the error of the side a consequent upon these,

$$d\, a = a\, d\, A\, (\cot. A - \cot. B),$$

which error becomes more minute, as A approaches to the ratio of equality with B.

If we assume the errors d A, d B as equal, but with different signs, or one in excess, the other in defect, then

$$d\, a = \pm\, a\, d\, A\, (\cot. A + \cot. B), \text{ but}$$

$$\cot. A + \cot. B = \frac{\sin. (A + B)}{\sin. A \sin. B}, \text{ and}$$

$$\sin. A \sin. B = \tfrac{1}{2} \cos. (A - B) - \tfrac{1}{2} \cos. (A + B),$$

the two measured sides, very nearly equal to two right angles. It is therefore evidently an "ill-conditioned" triangle, in which a small error in any of the measured quantities would cause considerable errors in the values deduced from them. Such a triangle would be inadmissible for an original base-line, which it would be preferable to form from only one of the measured lines; but, in bases of verification, such an arrangement as that adopted by Delambre must be occasionally resorted to in unfavourable localities, from the impossibility of otherwise connecting the stations, used in the general triangulation, with any measured test-line.

In general, no angle less than 30° should be used unless the nature of the localities should render its adoption necessary. If this condition be attended to, the accuracy of the determination of the calculated sides in a series of triangles will not fall much short of that which would be attained by actual measurement, were it practicable. For experience tends to prove that when all the triangles of a series are *well-conditioned**, the errors in the measure of the angles do not cause the consequent errors in the sides to accumulate through each successive step in the operation, but that they on the whole tend to compensate each other.

But in an extensive triangulation it is necessary that the sides of the primary triangles should be much longer

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Substituting, we have

$$\cot. A + \cot. B = \frac{\sin. (A + B)}{\frac{1}{2}\cos. (A - B) - \frac{1}{2}\cos. (A + B)}$$
$$= \frac{2 \sin. (A + B)}{\cos. (A - B) - \cos. (A + B)}$$

Substituting this last term in the value for $d\,a$, we have

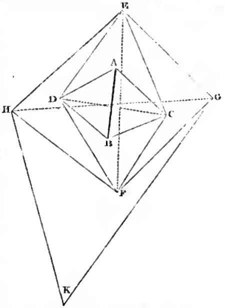
$$d\,a = \pm\, a\,d\,A \frac{2 \sin. C}{\cos. (A - B) - \cos. C};$$

hence $d\,a$ will be a minimum when $\cos. (A - B) = 1$, or $A = B$; for the larger the denominator $\cos. (A - B) - \cos. C$, the smaller will be the co-efficient of $a\,d\,A$. (See PUISSANT'S *Géodesie*, vol. i., page 136, and Notes to LESLIE'S *Trigonometry*, pages 471, et seq.)

* PUISSANT'S *Géodesie*, vol. i., page 136.

than the original measured base;—the sides of the triangles must therefore be increased without admitting any ill-conditioned triangles. This is accomplished with rapidity, as follows:—

A B is supposed to be the measured base, and C and D the nearest trigonometrical points*. All the angles being observed, the distances of C and D from the extremities of the base are obtained by calculation. In each of the triangles A D C, B D C, we then have two sides and the contained angle to find D C, one calculation acting as a check upon the other. This line D C is again made the base from which the distances,



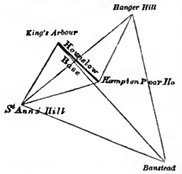
from D and C, of the trigonometrical stations E and F are computed; and the length of E F is afterwards obtained in the two triangles D E F and C E F. In like manner, the relative positions of H, G, K, &c., are obtained, and therefore, as we recede from the base, it will speedily become practicable to use, as bases, the sides of triangles much larger, and embracing much greater intervals. "Thus it becomes easy to divide the whole face of

* See Frome's *Trigonometrical Surveying*, page 14, and Sir J. F. W. Herschell's *Astronomy*, page 148.

a country into great triangles from 30 to 40, or even 100 miles in their sides, according to the natnre of the country. The vertices of these great triangles being once well determined, the country is afterwards, by a second series of subordinate operations, divided into smaller or secondary triangles, and these again into others of a still minuter order, till the final filling in is brought within the limits of personal survey and draftsmanship, and till a map is constructed with any required degree of detail."

In the late trigonometrical operations to constitute the frame work of the Irish survey, it was found advantageous to introduce triangles with sides from 70 to 90 miles in length:—in the triangulation carried on in the southern parts of England by General Mudge and Mr. Dalby, they deemed triangles whose sides were from 12 to 18 miles in length, preferable, for the general purposes of the survey, to triangles of greater dimensions.

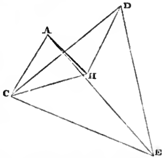
In the progress of their operations in the year 1792, a difficulty offered itself, when extending their triangles from the base on Hounslow Heath, according to the method of gradual increase above explained*.



The base, which had for its extremities King's Arbour and Hampton Poor House, having been measured, the next step would have been to form on each side the well-conditioned isosceles triangles, having respectively St. Ann's Hill and Hanger Hill as their vertices. From the calculated sides of these and their contained angles, a mean

* *Trigonometrical Surveying*, vol. i., page 31.

distance for a new and a longer base would have been obtained from St. Ann's Hill to Hanger Hill. But the triangle formed by Hanger Hill and the extremities of the base could not be completed, "because the station chosen at the former place being on the ground, there was scarcely a possibility of erecting a staff at King's Arbour, sufficiently high to afford a view of its top from Hanger Hill. As a proper substitute therefore, a station was chosen upon the elevated ground near Banstead, which was visible from St. Ann's Hill, King's Arbour, and Hanger Hill; and this, together with St. Ann's Hill and Hanger Hill, formed two triangles which would give the distance between St. Ann's Hill and Banstead independent of each other."



1. Thus in triangle A H C, we have the base A H, and the adjacent angles; hence we obtain the sides A C, C H.

2. In triangle A C E, we have the side A C, and the adjacent angles; hence we obtain the side C E.

3. In triangle C H D, we have the side C H, and the adjacent angles; hence we obtain the side C D.

4. In the triangle C D E, we have the side C D, and the adjacent angles; hence we obtain C E.

Exercises for Calculation*.

Given A H = 27404·2 feet, and the angles of the several triangles as below, required their sides.

Triangles.	Stations to which the Angles are referred.	Corrected Angles.	Lengths of the sides in feet.	Remarks.
A C H	A C H	74° 14′ 34″·5 44 18 51·75 61 26 33·75	A C = 34455·2 A H = 27404·2 C H = 37753·5	The length obtained at page 75 for A H was 27405·5347 feet, a value differing from that now adopted by nearly 1⅓ foot. In the reduction of the base giving the lesser value now adopted, account has been taken of the wear of the chains, which we neglected in the previous investigation, because it was irrelevant to the subject under consideration, namely, the reduction of the base to its value at a given temperature.
A C E	A C E	71 46 22 82 57 57 25 15 41	A C = 34455·2 A E = 80131·6 C E = 76687·7	
C D H	C H D	25 17 40·5 130 3 3·0 24 39 16·5	C D = 69278·3 C H = 37753·5 H D = 38670·0	
C D E	C D E	63 56 46·25 62 40 34·25 53 22 39·5	C D = 69278·0 D E = 77547·4 C E = 76687·7	

If for any cause it has been found advisable to commence the triangulation before the base has been measured, the sides of the triangles may be calculated from an assumed base, and afterwards corrected for the difference between this imaginary quantity and the real length of the base line. Or, as was found to be the case with one of the Indian bases, if the length of the base has originally been, from the want of access to correct standards, incorrectly reduced, the triangulation may be easily rectified;—the property of similar triangles readily points out the method to be employed.

The base measured near Gooty, for the Indian survey†, was found, after a careful comparison of the chains with the standard brass scale, to require a small correction. A standard chain referred to in that operation had been carefully tested some years before, and laid aside by Colonel Lambton, who relied on the belief that its length would

* *Trigonometrical Survey*, vol. i., pages 82, 83.

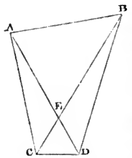
† *Asiatic Researches*, vol. xiii., and *Philosophical Transactions*, 1823.

remain invariable; however, a considerable period after the calculation of the triangles depending on the Gooty base had been completed, he was led again to test the length of the standard chain, and found that it must have been slightly too long at the time of the measurement of the base. This had led to erroneous results, most minute it is true, but still such as Colonel Lambton thought fit to notice and rectify. In the length of a degree, due to latitude 11° 37′ 49″, the required correction amounted to 1·25 fathom, the length of the degree having been originally calculated at 60480·3 fathoms, and the corrected length being 60481·55.

Reduction of Angles to the Centre of the Station.

In extending a series of triangles in populous neighbourhoods, wooded districts, or occasionally under other circumstances, instead of planting moveable signals at each point of observation, it will be found more convenient to select permanent well-defined objects, such as steeples, towers, windmills, &c., for the principal stations in the triangulation. But when a choice is made of such objects, the theodolite or circular instrument can seldom be placed in the centre or axis of the station. The observer, in such cases, approaches as near to the centre as he can with advantage, and calculates the quantity of error which the minute displacement may occasion. Thus, suppose it be required to determine the angle A C B which the remote objects A and B subtend at C, the centre of the permanent station; the instrument is placed in the immediate vicinity at the point D, and the distance D C with the angle A D C noted, while the principal angle A D B is observed. The central angle A C B may then be computed from the rules of trigonometry, but the calculation is effected by a simpler and a more expeditious method *.

* See Notes to Leslie's *Trigonometry*, page 469, and Puissant's *Géodesie*, pages 182, et seq.



Let C be the centre of the station, and D the axis of the theodolite. In the triangle A B C calculate the lengths A C, B C, from the base A B and the adjacent angles A and B.

The exterior angle of a triangle being equal to the two interior and opposite angles, we have,

$$A\,E\,B = A\,C\,B + C\,A\,D, \text{ and}$$
$$A\,E\,B = A\,D\,B + C\,B\,D; \text{ therefore,}$$
$$A\,C\,B + C\,A\,D = A\,D\,B + C\,B\,D, \text{ or,}$$

the angle sought,

$$A\,C\,B = A\,D\,B + C\,B\,D - C\,A\,D.$$

In the second term, the angles C B D and C A D are not given, we must therefore obtain a value for them in terms of known quantities, thus: in triangle C B D,

$$\sin.\ C\,B\,D : \sin.\ B\,D\,C :: C\,D : C\,B, \text{ whence}$$
$$\sin.\ C\,B\,D = \frac{\sin.\ B\,D\,C \,.\, C\,D}{C\,B}$$

And in triangle A D C,

$$\sin.\ C\,A\,D : \sin.\ A\,D\,C :: C\,D : A\,C, \text{ whence}$$
$$\sin.\ C\,A\,D = \frac{\sin.\ A\,D\,C \,.\, C\,D}{A\,C},$$

but the angles C B D, C A D being always very minute, the arcs may be substituted for the sines, and therefore

$$\text{angle } C\,B\,D = \frac{\sin.\ B\,D\,C \,.\, C\,D}{C\,B}$$
$$\text{angle } C\,A\,D = \frac{\sin.\ A\,D\,C \,.\, C\,D}{A\,C}.$$

Substituting these values of C B D and C A D in the above equation, we have

$$A\,C\,B = A\,D\,B + \sin.\ B\,D\,C \,.\, \frac{C\,D}{C\,B} - \sin.\ A\,D\,C \,.\, \frac{C\,D}{A\,C}$$

To adopt a more general notation,

Let l = station observed to the left,

r = station observed to the right,

O = observed angle,

y = angle subtended by l and centre of station,

then $O + y$ = angle subtended by r and centre of station,

Let d = distance to centre of station,

L = distance of station l from centre of station,

R = distance of station r from centre of station,

$$\left.\begin{array}{l}\text{The angle reduced}\\ \text{to the centre}\end{array}\right\} = O + \frac{d \,.\, \sin.\,(O + y)}{R} - \frac{d . \sin. y}{L}.^{*}$$

In applying this formula, attention must be paid to the signs of the sines of the angles, which will be negative when $(O + y)$ or y are greater than 180°.

Example. Let $O = 33° \ 58' \ 37{\cdot}43''$

$d = 3{\cdot}96$ yards

$R = 4510$ „

$L = 4730$ „

$y = 232° \ 55'$

then $O + y = 266° \ 53' \ 37{\cdot}43''$

Angle reduced to centre $= 33° \ 58' \ 37{\cdot}43''$

$$- \frac{(3{\cdot}96) \sin.\,(266° \ 53' \ 37{\cdot}43'')}{4510} + \frac{(3{\cdot}96) \sin.\, 232° \ 55'}{4730}$$

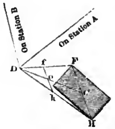
$O = 33° \ 58' \ 37{\cdot}43''$

Log. 3·96	0·597695				
Log. sin. (266° 53′ 37·43″) .	9·999361				
Colog. 4510 . . .	6·345824				
	6·942880	−		3	0·84
			33	55	36·59
Log. 3·96	0·597695				
Log. sin. (232° 55′) . .	9·901872				
Colog. 4730 . . .	6·325139				
	6·824706	+		2	20
Corrected angle			33	57	56·59

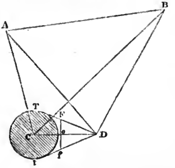
* *Opérations Géodesiques exécutées en Piémont et en Savoie:* Milan, 1827.

When the instrument is placed at the base of a tower or high permanent object, the centre of which is the true vertex of the triangle, it sometimes happens that the centre of the station cannot itself be seen, but the direction of that centre from the axis of the instrument is required for the purpose of measuring the angle A D C or y. The direction of the centre is found as follows:

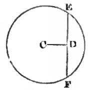
First, supposing the base of the station or signal to be rectangular; from the extremities of the diameter H F, draw the lines D F, D H, and measure their lengths; then on D F, take any point f and from D H, cut off D h, so that D F : D H :: D f : D h, then $f\,h$ is parallel to F H, one of the diameters. Bisect $f\,h$ in o, join D o; D o will be in the direction D C required.



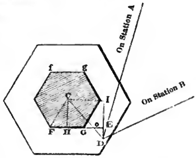
Supposing the base of the tower or signal to be circular: draw the tangents D T, D t, by sweeping the telescope of the theodolite round, until the visual ray describes a line touching the circumference of the tower. From D T, D t, cut off equal lines D F, D f, join F f, and bisect it in o, the line D o joining D and o will be in the direction of the centre C. The points F and f should be chosen as near as possible to the tower, in order that F f may be as long as possible.



If the instrument be placed within the circumference of the circle; through D, the axis of the instrument, draw the chord E F bisected in D. On D erect the perpendicular D C; D C will be the direction required.



If the position of the point D be such that both extremities of a diameter of the base cannot be seen from it, or if the base of the station be a regular polygon of any number of sides, the direction D C is obtained as follows:



From D draw D E perpendicular to F G or F G produced, measure D E. Bisect F G in H, and measure E H. Obtain the value of C H by measuring half the distance between the opposite sides F G, fg, or by the following proportion, in which the angle F C H is known, and depends on the number of sides of the regular polygon:

$$CH : FH :: \cos. FCH : \sin. FCH,$$

$$\text{hence } CH = FH\frac{\cos. FCH}{\sin. FCH}.$$

Then in the triangles D I C, D E o, we have

$$DI : IC :: DE : Eo, \text{ or}$$

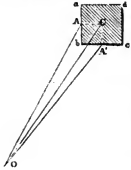
$$Eo = \frac{DE.IC}{DI}, \text{ from which equation}$$

the point of intersection of D C with F E, and consequently the direction D C, is obtained.

Form of Objects under Observation.

Attention is also to be paid to the form of objects or signals under observation. Those which do not terminate in a point, whether presenting the form of a truncated

pyramid, or a rectangular top, may lead to errors, when they are illuminated obliquely by the sun, causing thereby



the observer to direct his telescope not to the centre of the signal but to the centre of the face exposed to the light. For example, let $a\,b\,c\,d$ be the base of the signal observed from O. If, on account of the distance, the illuminated face $a\,b$ can alone be seen, the telescope will be directed to the point A, a middle point in $a\,b$ instead of the point C the centre of the signal, the amount of error being equal to the angle A O C. The value for this error is

$$A\,O\,C = \frac{A\,C \sin. A\,C\,O}{A\,O}$$

and the shorter the distance A O, provided it be great enough to prevent the sides in the shade from being seen, the greater will be the error. In very accurate trigonometrical operations, this correction is not to be neglected, as the error due to this cause has been known in cases of truncated pyramids with broad bases to amount to 15″ or 20″*.

When observations are made to churches, towers, or other permanent objects, it is desirable to make a slight sketch of their general form and appearance from the point whence they are observed; and if the signal be irregular in its outline, a mark is made showing the part of the object intersected, in order to avoid errors when the same object has to be viewed from another distant station, or if it should be necessary to re-observe the angles at a future time.

It is also desirable that a plan of the station of observation be made, marking the position of the axis of the instrument, with written dimensions of its distance by ordinates from some marked and fixed points. These data are useful to identify the station for future observations, and serve also when reducing angles to the centre.

Practical Directions on the Construction of Signals.

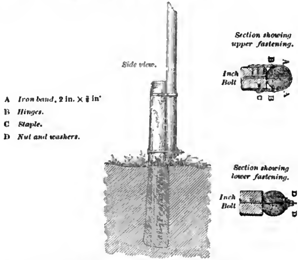
Permanent objects are, as we before observed, to be chosen in preference for signals: their advantages are solidity and consequent steadiness and durability; they also economize time and money otherwise expended in the special erection of signals. But such permanent objects do not always exist in localities best suited for stations, and they perhaps would form ill-conditioned triangles. They are in those cases to be determined in position by intersections, but special signals must be used for the summits of

* Delambre, *Base du Système Métrique*, vol. i., page 221.

the triangles. Even when permanent objects are applicable, they do not always dispense from the necessity or expediency of placing temporary signals on their summits, in order to render their intersection by the cross wires more precise.

To the variety of form and character of signals there is, of course, no limit; they depend on the nature of the country, its capabilities, the distances between the stations, the importance of the work, the outlay contemplated, &c. A few details, however, on signals adapted to different circumstances, may be given at this stage of our course.

In common surveys, embracing from several parishes to a whole county, as also in surveys for railroads, canals, and similar works, sides of triangles or connected bases have to be measured from 2 or 3, to 6 or 8 miles in length. For the purpose of ranging such lines, signals may be made by firmly fastening straight poles in the tops of high isolated trees, if their position gives the direction required.



An economical signal, suitable for lines of 8 or 10 miles in length, is represented in the above sketch, in which a long pole or mast, forming the signal, is held in a vertical

position by a strong post, the lower part of which is firmly fixed in the ground to a depth of 6 or 8 feet. A collar, towards the upper part of the post, confines the mast in its place, its lower end being fastened by a pin, round which it freely revolves as on a centre in a vertical plane, when the collar is unclasped or unlocked.

If the pole be very high and made of two pieces, or placed in an exposed situation, it is strengthened and kept upright by means of guys and stays. The advantage of this signal is, that it admits of the instrument being placed in the axis of the signal, which is for that purpose let down temporarily by being made to revolve on the pin that supports its base.

All posts or masts erected for signals are usually made to bear bunting flags; these should be of at least two colours,—red and white, when they are to be seen projected against trees or dark ground,—green and red, when they are to be relieved against the sky.

But as flags are useless in calm weather, a state of the atmosphere selected by preference for observations, they may be advantageously replaced by a small cone or barrel fastened towards the top of the pole. This may be painted red, if relieved against trees or ground; black, if relieved against the sky.

A good and easily recognised signal is also made by fixing at the top of the pole or mast a circular disk of sheet iron 2 or 3 feet in diameter, and a rectangular plate 4 or 5 feet long by 1½ broad; they are placed at right angles, one above the other. On their faces circular openings are cut, to diminish the surface of resistance offered to the air.

" The reflection of the sun from a plane mirror,

as affording a point of observation that might be seen at remote distances, was employed by General Roy in 1782, and in 1822 by Professor Gauss, while engaged in a trigonometrical measurement in Hanover; and the principle was adopted in this country by Colonel Colby and Captain Kater, when verifying General Roy's triangulation connecting the meridians of the Greenwich and Paris observatories. At their concluding station on Shooter's Hill, seven or eight days elapsed, during which Hanger Hill tower, though only 10 miles distant, remained completely obscured by the dense smoke of London*." To overcome this cause of delay, several flat plates of polished tin were attached one below the other to the signal post, at angles calculated to reflect the sun's rays in the required direction, the inclination of the plates being so computed in reference to the relative position of the two stations and the sun that they should keep up a tolerably continuous reflection for a considerable time, the rays being caught and thrown from each plate nearly as soon as by the motion of the sun they had left the plate above it†. This plan was successful, and in subsequent operations in 1823 it was again resorted to, and with equal success.

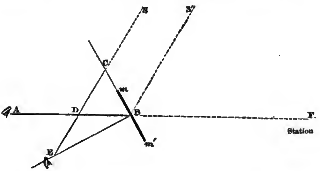
"The utility of employing the sun's reflection, as a point of observation, being established by the result of these experiments, an instrument was constructed" by Lieutenant Drummond, R.E., "capable of adjustment, that might be used on all occasions, and easy of management."

A B is the axis of a telescope, directed towards the station F, to which the sun's rays are to be reflected; if the station itself should be invisible, the direction is ascertained relatively to some nearer object. A mirror $m\,m'$ is attached to a bar B C, which revolves round a universal joint at B; and the mirror is adjusted with its plane at right angles to

* *Philosophical Transactions*, 1826.

† *Memoir of the Professional Life of Captain Drummond*, by CAPTAIN LARCOM, R.E., vol. iv., *Roy. Eng. Papers*.

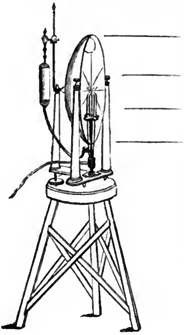
a plane passing through A B and the middle of the mirror B C. A bar B E is fixed at right angles to the plane of the mirror, and a bar E D C revolves round the point D in the plane E B C; while its extremities rest on the bars B C, B E; D being the centre of a circle passing through the three points E, B, C, and D B being therefore always equal



to D C. A small telescope, provided with a rectangular eye-piece, is fixed to the bar E C, and serves to direct it towards the sun, and to follow its course. When the telescope or bar is so directed, the rays of the sun will be reflected to the distant station, in lines parallel to the axis of the telescope A B; for, the angle *m'* B F is equal to the angle D B C = angle D C B by construction = angle C B S' (the rays from the sun being parallel); therefore the angle C B S' is equal to the angle *m'* B F; but, S' B being the ray of incidence, and B F the ray of reflection, the reflected ray will proceed in the direction B F.

"General Roy had, on several occasions, but especially in carrying his triangles across the channel to the French coast, made use of Bengal and white lights; for these, parabolic mirrors, similar to those with which our light-houses are supplied, and illuminated by argand burners, were afterwards substituted," as the short duration of the first rendered them inconvenient. This power again

proving inadequate to the purposes of subsequent operations, in 1822, Colonel Colby and Captain Kater, conjointly with MM. Arago and Mathieu, employed, for the first time, an apparatus of a different kind. A large plano-convex lens, 0·76 metre in diameter, was substituted for a parabolic reflector, and the illuminating body used was an argand lamp with four concentric wicks. The lens was composed of a series of concentric rings reduced in thickness, and cemented together at the edges. The light which this gave is stated to have been 3¼ times the intensity of that given by a reflector, appearing at the distance of 48 miles like a star of the first magnitude*.



"But valuable as this apparatus may be when employed in a light-house (the purpose for which it was invented and constructed by M. Fresnel), the properties of the parabolic reflector appeared still to give it a preference for the service of the trigonometrical survey, provided a more powerful light could be substituted in its focus, instead of the common argand lamp." This object was accomplished by Lieutenant Drummond, by submitting a ball of chalk-lime to a stream of oxygen directed through the flame of alcohol. The

* *Philosophical Transactions*, 1826, p. 328; and 1828, p. 154.

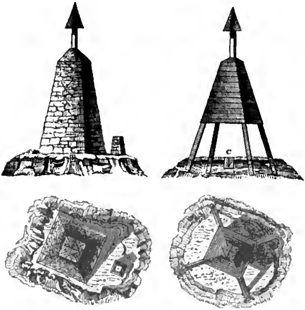
size to be given to the ball of lime is regulated by the amount of divergence required to be given to the light, the rays only that proceed from the focus being reflected in a direction parallel to the axis of the parabola. The diameter generally used by Lieutenant Drummond was $\frac{1}{4}$ inch, "as it proved quite sufficient to make the requisite allowance for aberration in the reflector from its true figure, as well as uncertainty of direction arising from terrestrial refraction." The light emitted was estimated from the mean of ten experiments as possessing 83 times the intensity of the brightest part of the flame of an argand burner of the best construction, and supplied with the finest oil.

The first application of the lamp to actual use took place in Ireland in 1825. A station on Slieve Snaght, in Donegal, an important point in the triangulation which connects the North of Ireland with the Western Islands of Scotland, had been looked for in vain for upwards of two months from Divis Mountain, near Belfast, the distance being 66 miles. Colonel Colby resolved that an attempt should be made to surmount this obstacle by the instrument described. Its direction was referred at night to a 15-inch parabolic reflector, illuminated by an argand lamp, and placed as a guide nearly in the same direction. The light on the distant signal was not only seen with the naked eye, but appeared much larger and brighter than the guiding light, their relative distances being 66 and 15 miles.

In mountainous districts the summits of the mountains are selected as the sites of the stations; and the signals may be made of pyramidal heaps of stone (or "cairns"), raised over a permanent mark fixed in the ground to denote the axis of the station, and over the centre of which a pole similar to those previously described is kept steady and upright by the stones surrounding its base.

In very elevated situations, where the signals are exposed not merely to the influence of storms, but also to the destroy-

ing effects of alternating snows and frosts, it may be desirable in national surveys to give such signals greater stability. The annexed figures represent the signals used in a triangulation carried across the Alps*, in which some of the stations were at a greater elevation than the glaciers, and on steeps acces-



sible only to men of firm nerves. When made of wood of the form represented, no reduction to the centre was required; when built of solid rubble work, a small pillar was erected by their side, to serve as a stand for the instrument, to which it afforded a more stable basis than the common wooden tripod, which is liable to yield and alter its form under every blast. Before the signal was built,

* *Opérations Géodesiques, exécutées en Piémont et en Savoie;* Atlas. Milan, 1827.

the distance of the centre of the station from the point selected for the axis of the instrument was carefully measured.

To discover Lost Stations.

In the progress of the triangulation carried on in 1821 by Colonel Colby and Captain Kater, for the purpose of connecting geodesically a second time the Paris and Greenwich meridians, it was found necessary to refer to the station at Fairlight, near Hastings, used in General Roy's previous triangulation. All trace of it was effaced on the ground, but the distance of the station from the centre of a mill in its vicinity having been registered by General Roy, Captain Kater thus describes the method adopted to find it*.

The distance from the centre of the mill was used as a radius, and in the segment of the circle described, a small theodolite was moved until the surrounding stations were found to subtend the angles registered. After the station had been thus discovered, the permanent bench mark, that had been fixed by General Roy, was found on sinking below the surface, its central point being precisely underneath the axis of the theodolite.

In this case, the distance of the station from a known point being given, the problem was of easy solution;—in cases where no such record has been preserved, the centre is found according to the following method, devised by Colonel Colby†.

Let D be the lost station, the position of which is required, and A, B, C, G, the surrounding stations observed from D. Assume T as near as possible to the supposed site of the point in question, and take the angles A T B,

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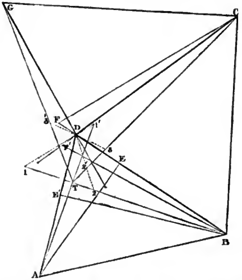
* *Philosophical Transactions*, 1828, p. 155.

† Frome's *Trigonometrical Surveying*, p. 24.

B T C, C T G. If the angle A T B be less than the original angle A D B, the point T is evidently without the circle in the segments of which A, B and D are situated (Euc. 32, I.); if the angle be greater, it is, of course, within the segment. The same holds good with respect to the angles B T C and B D C, C T G and C D G.



Recompute the triangle A B D, assuming the angle at D to have been so altered as to have become equal to the angle at T, and that the angle at A is the one affected thereby: under that supposition the point T will be in B D or B D produced.

Again, recompute the triangle, supposing the angle at B the one affected, in which case the point T will fall on A D or A D produced.

In like manner, in the triangles B D C, C D G, recompute the triangles, supposing the angles at B, C and G to be alternately affected by the changes in B D C, C D G. These computations will give the triangles A B E, A B E′,

B C F, B C F′, &c., calculated with the values of T, as observed at the first station, and the angles at A, B, C and G alternately changed by the amount of the difference between D and T.

From the point T, on A T and B T, or on A T and B T produced, set off T 1, T 1′, respectively equal to E D and E′ D, the differences between the distances just found and the true distances of A and B from the point D. If the true angle D be smaller than the approximate angle T, T 1 and T 1′ are set off on A T and B T produced. Join the points 1 1′. A repetition of the same process in the triangle B C D gives the point 2 2′, to be joined also by a straight line: finally, the same operation with respect to the triangle C D G gives the points 3 3′, which are also joined by a straight line. The common point of intersection of these three lines marks the centre of the station required. If they should not meet in a common point, but form a small triangle, the middle of that small triangle will give the station required; or, if on the observation of the angles subtended by the surrounding stations at that point, some difference should be yet found, this point is then assumed as a second nearer step in the approximation, and the same process repeated with the new angles obtained. It rarely happens, however, that more than the first operation is required.

"To save computation on the ground, it is advisable to calculate previously the difference in the number of feet that an alteration of one minute in the angles at A, B, C, and G, would cause respectively in the sides B D, A D, C D, and D G. The quantities thus obtained being multiplied by the errors of the angle at T, will give the distances to be laid off from T in the directions B T, A T, &c."

Forms of Entry for Horizontal Angles.

When no repetition is made in the reading of the angles, they may be entered in the field-book according to the annexed form.

Observations at East end of Base on Putney Heath. June, 1841.					
Name of Station.	Vernier A.	Vern. B.	Mean reading.	Right or left of first object.	Remarks.
Chimney, Heathfield-house	70° 19′ 0″	18′ 40″	70° 18′ 50″	..	Cloudy, and light wind.
Wimbledon Windmill	103 32 20	32 40	103 32 30	R.	
Combe Wood Telegraph ..	115 14 20	14 50	115 14 35	R.	Sketches of Stations in field-book A, page 4.
West end of Base	159 30 20	30 20	159 30 20	R.	
Finger-post, cross-roads ..	194 57 0	57 10	194 57 5	R.	
Putney Heath Telegraph..	224 9 40	10 0	224 9 50	R.	
Chimney, Heathfield-house	70 19 0	18 40	70 18 50	R.	

The entries in the fifth column, denoting whether the signal is read to the right or left of that immediately preceding, facilitate the recognition of the objects at any future time.

When multiples of the angles are taken by the method of repetition explained in page 49, and when great care is otherwise bestowed on the observations, they are conveniently entered according to the following form, which was that adopted in the Piedmontese triangulation*, and in which the state of the barometer and thermometer is also registered, as the state of the atmosphere influences the amount of refraction, a subject of which we shall treat in the next chapter.

* *Opérations Géodesiques exécutées en Piémont et en Savoie:* Milan, 1825.

Angles between the Signals, Avebury (left), Long Knoll (right).
June 26, 1828, 10 a.m. First Series.

No. of repetitions.	Multiple arcs read from Vernier A.	Single.	Verniers and Means.			Remarks.
			Vernier	First and last reading.	Means.	
	° ′ ″	′ ″		° ′ ″	° ′ ″	
1	76 40 20	40 20·0	A.	0 0 0	46 43 20	Barometer 29·276.
2	153 20 40	„ 20·0		46 43 20		
3	230 1 0	„ 20·0	B.	90 0 10	„ 43 20	Attached Thermometer 61°.
4	306 41 20	„ 20·0		43 30		
5	23 21 40	„ 20·0	C.	180 0 10	„ 43 20	Detached Thermometer 57°.
6	100 2 0	„ 20·0		43 30		
7	176 42 20	„ 20·0	D.	270 0 0	„ 43 10	Calm and clear.
8	233 22 40	„ 20·0		43 10		
9	330 3 0	„ 20·0			4) 70	
10	46 43 20	„ 20·0		Mean	46 43 17·5	

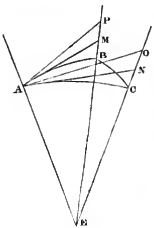
Twice the circumference + 46° 43′ 17·5″ = 766° 43′ 17·5″
Mean angle .. .. = 76° 40′ 19·7″

Triangles, not Plane, but Spherical.

With respect to the angles thus observed, and the triangles combined from them, they are not, rigorously speaking, *plane*, but *spherical*, existing on the surface of a sphere, or rather, to speak correctly, of a spheroid. In small triangles, of six or seven miles in the sides, this consideration may be neglected, as the difference is imperceptible; but in larger ones it must be taken into consideration.

"It is evident that as every object used for pointing the telescope of a theodolite has some certain elevation, not only above the soil, but above the level of the sea; and as, moreover, these elevations differ in every instance, a reduction to the horizon of all the measured angles would appear

necessary. But in fact, by the construction of the theodolite, which is nothing more than an altitude and azimuth instrument, this reduction is made in the very act of reading off the horizontal angles.



"Let E be the centre of the earth; A, B, C, the places on its spherical surface to which the three stations, A, P, O, in a country are referred, by radii E A, E B P, E C O. If a theodolite be stationed at A, the axis of its horizontal circle will point to E when truly adjusted, and its plane will be a tangent to the sphere at A, intersecting the radii E B P, E C O, at M and N, above the spherical surface. The telescope of the theodolite, it is true, is pointed in succession to P and O; but the readings of its azimuth circle give, —*not* the angle P A O between the directions of the telescope, or between the objects P, O, as seen from A, but the azimuthal angle M A N, which is the measure of the angle A of the spherical triangle B A C. Hence arises this remarkable circumstance, that the sum of the three observed angles, of any of the great triangles in geodesical operations, is always found to be rather more than 180°*." Ramsden's large theodolite, three feet in diameter, was the first instrument by which this excess, called the spherical excess, was observed. It is always a minute quantity, seldom exceeding 4″ to 5″, in the triangles used in geodesical operations.

* Sir J. F. W. Herschell's *Astronomy*, p. 149.

" Were the earth's surface a plane, the sum of the three angles would be exactly 180°; and the excess above 180° is so far from being a proof of incorrectness in the work, that it is essential to its accuracy, while it offers at the same time another palpable proof of the earth's sphericity.

" The true way then of conceiving the subject of a trigonometrical survey, when the spherical form of the earth is taken into consideration, is to regard the net-work of triangles into which the country is divided, as the bases of an assemblage of pyramids converging to the centre of the earth. The theodolite gives us the true measures of the angles included by the planes of those pyramids; and the surface of an imaginary sphere, at the level of the sea, intersects them in an assemblage of spherical triangles, above whose angles, in the radii prolonged, the real stations of observation are raised by the superficial inequalities of mountain and valley." When the triangles, therefore, are large enough to make the difference between the arcs and their chords sensible, the condition of sphericity possessed by these triangles must be taken into consideration: otherwise, were the whole extent of the triangles stretched out on a plane, erroneous results would arise, increasing in magnitude as the series of triangles extended.

Theso triangles would therefore be naturally treated as spherical triangles in calculating their sides; but because triangles, which are the subject of consideration in geodesic operations, differ very little from plane triangles, their sides being very small with respect to the radius of the sphere, the calculations may be simplified by treating them as plane triangles, the angles of which are corrected by means of Legendre's well-known theorem *, viz.:

" If there be a spherical triangle, the sides of which are very small compared with the radius of the sphere, it may be considered equivalent to a plane triangle, which has its

* THOMSON'S *Trigonometry*, p. 93.

sides equal to the sides of the proposed triangle, and its angles eqnal to the angles of the same diminished respectively by one-third of the spherical excess."

The excess of the sum of the three observed angles above two right angles might therefore be employed, one-third of such excess being deducted from each angle of the triangle, and the calculations made as with a plane triangle. But in order to detect any errors that may have occurred in the observation of the angles, the area of the triangle is calculated, (with sufficient accuracy, by considering it for the time as a plane triangle,) and from the area thus obtained the true spherical excess is deduced. Then, if the angles have been observed correctly, their sum ought to exceed 180° by the spherical excess obtained; any difference will be the error of observation; and each angle may be corrected by a third of this error, unless the observer should have cause to donbt the accuracy of one more than another, in which case, he may distribute the error in the proportion that he deems most applicable to the particular case.

The area of the triangle, from which the spherical excess is to be deduced, is itself obtained thus: (see note, page 30.)

If two sides b and c, with the contained angle A, be given, then the

$$\text{Area } S = \tfrac{1}{2}\, b\, c \sin. A.$$

If one side a, and the two adjacent angles B and C, be given, the

$$\text{Area } S = \tfrac{1}{2}\, a^2 \frac{\sin. B \sin. C}{\sin. (B + C)}.$$

Denoting the spherical excess by e, then, because the spherical excess is proportional to the area of a spherical triangle*,

$$e = \frac{S}{r^2};$$

* THOMSON's *Trigonometry*, p. 53.

r being the radius of the earth expressed in the same unit as the sides of the triangle; and if we make R'' = the number of seconds comprised in the radius, we shall have the excess in seconds expressed

$$e \text{ (in seconds)} = \frac{S}{r^2} \cdot R''.$$

Therefore, in order to obtain the spherical excess in seconds, the (log. R'' − 2 log. r), a constant quantity is added to the logarithm of the area of the triangle.

If the sides of the triangle be expressed in yards, the

Log. of r = 7,002,667 will be = 6·84526, and
Log. of R'' = 206,264·9 „ = 5·31443,

whence log. R'' − 2 log. r = (1·62391 − 10), a constant quantity to be added to the logarithm of the area of the triangle to obtain the logarithm of the spherical excess in seconds.

*Example**. Let a = 82743·3 yards, b = 70877·8 yards, and C = 103° 19′ 10″.

Then, to calculate the area of the triangle:

$$\text{Area} = \tfrac{1}{2}\, a\, b \sin. C.$$

Log. a . .	4·917733
Log. b . .	4·850510
Log. sin. C	9·988135
Colog. 2 .	9·698970
	9·455348 = log. of area.
Constant logarithm −	9·623909
	1·084492 = log. of 12·15″ = excess.

The spherical excess in this case is very great, owing to the large size of the triangle: in common operations it seldom exceeds 4″ to 5″.

* MALORTIE'S *Topography*, p. 93.

*Exercises for Calculation**.

One side and adjacent angles given, hence

$$S = \frac{1}{2} a^2 \frac{\sin. B \sin. C}{\sin. (B + C)},$$

in which $a = 32608{\cdot}64$ feet.

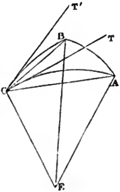
Vertices of the Triangles.	Observed Angles.	Spherical excess.	Error.	Angles for calculation.	Distances in feet.
A. Gootydroog Station	° ′ ″ 65 18 41·19			° ′ ″ 65 18 41·10	
B. North end of base	87 27 16·45			87 27 16·36	
C. South end of base	27 14 2·64			27 14 2·54	
	180 0 0·28	0·13″	+0·15	180 0 0	
Gootydroog Station from North end of base ..					16423·9
Gootydroog Station from South end of base ..					35853·8
B. North end of base	35 4 2·45			35 4 2·09	
C. South end of base	105 3 6·43			105 3 6·07	
D. Paumdy-hill Stat.	39 52 52·21			39 52 51·84	
	180 0 1·09	0·22	+0·87	180 0 0	
Paumdy-hill Station from North end of base ..					49111·3
Paumdy-hill Station from South end of base ..					29218·8

Delambre, in the geodesical operations forming the basis of the metrical system, adopted, instead of this method, the process of reducing the horizontal angles, comprised between the objects, to the angles comprised between the chords of the arcs: and with the assistance of the formulæ which he prepared for the practical solutions, his method was found so rapid in the calculations, that it has been generally adopted. On the English Trigonometrical Survey, the triangles have thus been calculated as if they were formed

* Colonel Lambton's Account of the Indian Survey, *Asiatic Researches*, vol. xiii., p. 33.

by right lines supposed to join the projection of those stations on a common horizontal basis at the level of the sea.

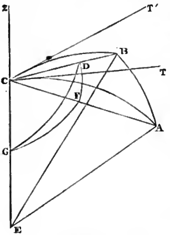
The problem of reducing the spherical angle contained between two terrestrial objects to the angle of the chords, resolves itself into the reduction of the angle, formed by the intersection of two planes measured on a third intersecting plane, to the value of the angle with the same vertex, but measured in another intersecting plane inclined to the first.



In the annexed figure let E be the centre of the earth, C the station of the observer at which the angle, subtended by the stations whose projections on the sphere at the level of the sea are A and B, is measured. The angle taken by the theodolite being the horizontal angle is represented by T C T′, T C and T′ C being tangents at C to the arcs of the great circles A C, C B. The angle TCT′ is in fact the angle formed by the planes A C E, B C E, at their common intersection, the radius C E. The angle made by the chords is the angle A C B, which angle is smaller than T C T′, and the more the plane of the chords represented by A C B inclines towards E, the smaller will the angle A C B become, until it would vanish altogether when the plane came to coincide with CE. From this we draw the conclusion that the larger the triangle the greater is the difference between the horizontal angle formed by the tangents to the arcs, and the angle formed by the chords of the arcs. This difference is obtained as follows: the angle of depression,

$T C A = \frac{1}{2}$ arc C A, and

$T' C B = \frac{1}{2}$ arc C B. (Euclid, III. 20 and 32.)



From C, with a radius equal to unity, describe the arcs D G, D F, F G, intersecting the chords and the earth's radius C E; then we have a spherical triangle D G F, in which are given the angle at G, or the angle of intersection of the planes,—*i. e.*, the angle T C T′, also

$$\text{the side G D} = 90^\circ - \text{angle T}' \text{C B},$$
$$= 90^\circ - \tfrac{1}{2} \text{ arc C B, and}$$
$$\text{the side G F} = 90^\circ - \text{angle T C A},$$
$$= 90^\circ - \tfrac{1}{2} \text{ arc C A}.$$

The arcs C B, C A, are obtained by approximation, calculating the triangle as if it were a plane triangle, and converting their values into degrees, minutes, and seconds.

From these data, two sides and the contained angle, the arc D F is obtained, but

$$\text{D F} = \text{angle A C B} = \text{angle required.}$$

The sum of the three angles in a triangle, thus reduced, should be equal to 180°; any difference, whether in excess or defect, is the amount of error, and is distributed among the three angles.

Delambre investigated a formula, and prepared corresponding tables, by the assistance of which these calculations are effected with great rapidity*.

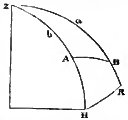
* The investigation of this formula is given in PUISSANT'S *Géodesie*, vol. i., p. 107.

Exercise for Computation.

Vertices of the Triangles.	Observed Angles.	Spherical excess.	Error.	Angles for calculation.	Distances in feet.
Dean Hill	62° 22′ 48″·75			62° 22′ 47″	
Butser Hill	48 28 41·5			48 28 40	
Highclere	69 8 35·			69 8 33	
	180 0 5·25	4·07″	+1·18	180 0 0	
	Butser Hill to Highclere				148031
	Dean Hill to { Butser Hill				156122
	Dean Hill to { Highclere				125085

REDUCTION TO A HORIZONTAL ANGLE OF AN ANGLE OBSERVED BETWEEN TWO OBJECTS SITUATED IN A PLANE OBLIQUE TO THE HORIZON.

When, instead of the theodolite, the sextant or repeating circle is used to measure the angular distance between two objects, the horizontal angle or angular distance of the projections of those points on the plane of the horizon is obtained thus:



Let A and B be the objects observed, and H, R the projections of A and B on the plane of the horizon, required the arc H R. Observe the angular elevations A H, B R, as explained in the description of the Sextant.

H R is the same as the vertical angle Z in the spherical triangle Z A B, of which the measured angle A B is the

base, and the complements of the elevations the sides. The vertical angle is therefore obtained by the formula

$$\text{Sin.}\ \tfrac{1}{2}\ Z = \sqrt{\frac{\sin.\ (s-b)\ \sin.\ (s-a)}{\sin.\ b\ \sin.\ a}}$$

in which s is equal to half the sum of the sides.

On the Continent, where Borda's repeating circle is in almost universal use, every angle observed between terrestrial objects has to be reduced to the horizon: a work of endless labour if a simpler formula than the above had not been found. Considering the triangle as one composed of two sides differing little from a quadrant, a formula less complex is obtained, and from it tables have been calculated which give very rapidly the reduction required. The investigation of the formula will be found in PUISSANT'S *Géodesie*, vol. i., page 109.

BASES OF VERIFICATION TO BE MEASURED.

The angles of a series of triangles having been observed, and the sides calculated from independent data to prove their accuracy, an additional test is adopted in the actual measurement of one or more of the distant sides to serve as bases of verification; these sides being, of course, measured with the same care as it was deemed expedient to bestow on the measurement of the original base. The accuracy of the work having been thus ascertained, the next step consists in the plotting or protracting of the triangles to the scale determined upon for the survey.

It is customary to plot all surveys, but with large surveys of counties or of kingdoms, it is the universal practice, to plot them with the north upwards. For this purpose, and under any circumstances, the direction of the meridian with reference to the triangulation is to be laid down. It is necessary, therefore, to observe the direction of the meridian with respect to the original base, or some

one of the sides of the principal triangles, from which the azimuthal distance of each part is given. When treating of the subject of longitude, we shall give an account of the methods that may be adopted to ascertain with precision the direction of the meridian at any one station; we shall here describe some ready methods by means of which its direction can be obtained with approximate accuracy.

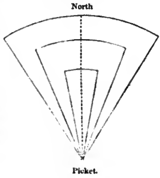
Meridian Line*.

Fix the theodolite at one of the stations used in the triangulation, and some hours before mid-day direct the telescope so that the cross wires shall touch the upper or lower limb of the sun in the east; note the horizontal and vertical readings of the arcs;—repeat the operation at short intervals, taking care to direct the intersection of the cross wires to the same limb of the sun that was before observed, and note all the readings in their regular succession.

Again, in the afternoon, when the sun descends westwards, clamp the vertical arc to the last reading, and note

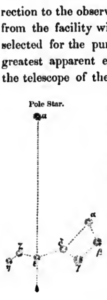
* When no angular instrument is at hand, an approximate meridian line may be set out as follows:—

Drive a thin staff or picket vertically on a level piece of ground, a gravel walk for instance. Several hours before noon measure the length of shadow thrown by the picket, and from the base of the staff as a centre, with the length of the shadow as radius, describe an arc of a circle from west to east. About the same interval of time after mid-day, observe the point where the extremity of the shadow again coincides with the arc; a line drawn from the centre of the staff to the middle point of the arc thus intersected, will be nearly in the direction of the meridian. It will be better to describe three or four such arcs at different elevations of the sun, and to make use of the mean of their central points to trace the meridian line.



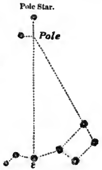
the horizontal angle at the time of the sun's limb touching the intersection of the cross wires. The vertical arc being clamped in succession in the descending series of the vertical angles, all the horizontal readings at the time of each successive intersection are entered. The point on the horizontal limb half way between all the readings will give the angle to which the vernier is to be placed, in order that the telescope may point to the position occupied by the sun at noon. A picket driven into the ground in that direction serves to mark the meridian line, and the angle, formed between it and any side of the triangles having the selected point for a vertex, being taken, the azimuthal direction of each and all the sides of the triangles is obtained. This method would be quite correct if the sun moved constantly in the same parallel, but the change in his declination between the time occupied by the observations renders necessary a correction to be explained hereafter.

The same method, however, is applicable without correction to the observation of a fixed star, and the pole star, from the facility with which it is identified, is frequently selected for the purpose, being observed at the time of its greatest apparent eastern and western elongation. But if the telescope of the theodolite be not powerful enough to observe the star under these conditions, (as one of the observations must generally be made by day light,) a very close approximation may be had by remembering that the pole star very nearly reaches the true meridian, when it is in the same vertical plane with the ϵ or Alioth, or the star in the tail of the Great Bear, which is nearest to the quadrilateral. The vertical position can be ascertained by means of a plummet. To see the cross wires in the field of the telescope at the



same time with the star, a faint light should be placed near the object glass. When the pole star has been brought correctly into the central part of the angle formed by the intersection of the cross wires, the horizontal limb is firmly clamped, and the telescope brought down to the horizon; and a light, seen through a small aperture in a board, and held at some distance by an assistant, is moved according to signals, until it is bisected by the wires. A picket driven into the ground underneath the light serves to mark the meridian line for reference by day, when the angle formed between it and the side of the triangle may be measured.

The true situation of the North Pole may also be nearly ascertained by the following indications of the stars near it. In the first place, as shown above, a straight line drawn from the pole star to the star Alioth, or ε, in the Great Bear, passes through the pole, and a perpendicular to this line at the pole passes through the small star nearest the pole. Finally, the stars called the "Pointers," in the above-named constellation, point almost directly towards the pole. The pole star is distant from the pole about 2⅜ degrees.



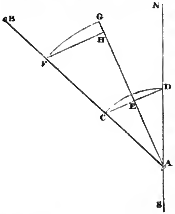
Of Protracting the Triangulation.

In protracting the triangulation, it is better to lay down the triangles from the lengths of their sides, than by measuring the angles; because measures of length can be taken from a scale, and transferred to the plan with more exactness than angles can be pricked off from a protractor. Beam compasses, with vernier scales attached, are used in this operation. Bearing in mind this precaution, it will be necessary to obtain the direction of the meridian by some

other means than by simply laying down the angle it forms with the station observed, in order to preserve a uniform degree of exactness in each part of the work. The meridian must also, therefore, be plotted by means of measures of length. A ready way offers itself, by calculating the lengths of the sides in a right-angled triangle, having for one of its angles the azimuthal distance of the observed side, and the said side for the hypotenuse.

But the following method described by Mr. Simms* will be found convenient, at the same time that it serves to illustrate a useful application of the table of natural sines.

Let A B be the side of the triangle, the azimuthal angle of which has been ascertained with reference to N S, the meridian line. Take from an accurately divided diagonal scale, exactly 5 inches as a radius, and from A, as a centre, describe an arc C D; now the chord of an arc being equal to twice the sine of half the arc, the chord C D is equal to twice C E, the sine of half the angle C A D. Take a radius A F equal to twice A C, and describe the arc F G intersecting the radius A B in F, draw the sine F H, then by similar triangles,



A F : A C :: F H : C E, but

A F = 2 A C by construction, therefore

F H = 2 C E = C D;

that is, the chord of a given arc is equal to the sine of half the arc with double the radius.

* Simms on *Mathematical Instruments.*

The radius of the tables of natural sines is equal to 1 or 10; and having taken the half of 10 or 5 inches for the radius A C, the natural sine of half the given angle taken from the tables will correspond to F H, the sine of half the given angle with double the radius; but F H was proved equal to C D; the natural sine therefore of half the given angle to a radius 10, will be equal to the chord of the whole angle to a radius 5. Having taken that distance from the same scale of inches as the radius, place one foot in the point C, and with the other mark the point D on the arc C D, then through D and A draw the the line N S, which will be the direction of the meridian.

When the operations of a Trigonometrical Survey are extended, in eastern or western directions, beyond spaces of about 60 miles from a fixed meridian, it is expedient to observe new meridians, in order to avoid errors which would otherwise take place as the result of computations made on the supposition of the earth's surface being a plane. Within a limit of about 60 miles such a supposition produces no sensible error*.

Understanding the above application of a table of natural sines, it will be seen how it may be used to divide with accuracy a circle or circular protractor, on a piece of card-board, by using 5 inches for the radius of the circle, and reading the chords of arcs submultiple of the circumference from the tables, measuring them with care on a diagonal scale.

When the skeleton triangulation is completed, the plan is ready to receive the delineation of the roads, streams, legal and ecclesiastical boundaries, and whatever detail is included under the general head of interior filling in of the map.

Before giving practical directions for these operations, we must consider the subject of levelling; levelling operations forming an essential part of a trigonometrical survey.

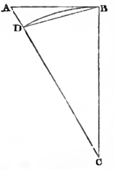
* *Trigonometrical Survey*, vol. ii., page 4.

CHAPTER IV.

ON LEVELLING AND REFRACTION.

Levelling is the art of finding a line parallel to the horizon at one or more stations, in order to assign the difference of altitude between one place and another. "Two or more places are on the same level, when they are equally distant from the centre of the earth. Also, one place is higher than another, or above the level of it, when it is further from the centre of the earth; and a line, equally distant from that centre in all its parts, is called a line of true level. Hence, because the earth is round, that line must be a curve, and make part of the earth's circumference, or at least be parallel to it*."

But the lines of sight which determine relative levels cannot evidently trace a curve parallel to the earth's surface, and a horizontal line can be traced only by a series of right lines tangent to the earth's surface approximating more nearly to a line of true level, the shorter the sides of the circumscribing polygon are chosen.



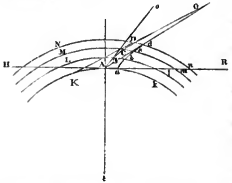
Let the arc B D be a portion of the earth's surface whose centre is C; and let the tangent A B, horizontal at B, meet the vertical line C D in A. The line B A will be the apparent line of level, and the arc B D the true line of level from the point B, and at any point D, A D is the height of the apparent above the true level. This difference, it is evident, is always equal to the excess of the secant of the arc B D above the radius of the earth.

* Hutton's *Mathematical Dictionary*.

The quantity of depression, A D, is easily computed; for $AB^2 = (2\,BC + AD)\,AD$ (Euc. III. 36), or very nearly $= 2\,BC \,.\, AD$, hence $AD = \frac{AB^2}{2\,BC}$. As 2 B C, the diameter of the earth, is a constant quantity, the depression is proportional to the square of the distance. In the space of one mile, this depression will amount to $\frac{5280}{7916}$th parts of a foot,—and from this we derive an easily remembered formula for the approximate correction for curvature, which may be expressed in fact by two-thirds of the square of the distance in miles.

But this effect due to the earth's curvature is modified by another cause arising from optical deception. Experience has shown that rays of light, in passing obliquely from a medium of a given density into another of greater density, change their direction, and approach more nearly to that of a perpendicular raised to the common surface at the point where they enter the denser medium. Now, the atmosphere increasing gradually in density from its external limits to the surface of the earth, may be supposed to consist of successively superposed minute layers, each concentric with the general surface of the sea, and each of which is more rarefied, or specifically lighter, than that immediately beneath it; and denser or specifically heavier than that immediately above it. A ray of light, therefore, passing obliquely through the atmosphere, for example from a higher to a lower level, to the eye of an observer, passes from the rarer to the denser strata; and following the above law of optics, it will be diverted from its original course, and made to approach more and more nearly to a perpendicular to the horizon. It will thus describe a curve concave to the earth's surface; but it is a law in optics that an object is seen in the direction which the visual ray has on arriving at the eye, without regard to what may otherwise have been its course between the object and the eye: the object appears, there-

fore, in the direction of the tangent to this curve. This optical effect or apparent displacement of the object is called *refraction*. Every difference of level, accompanied as it must be, with a difference of density in the strata of the atmosphere, will have, corresponding to it, a certain amount of refraction, and as the curve described by each ray of light is concave next the earth, the tangent to the curve will lie above it, and consequently the object will appear more elevated above the horizon than if there were no atmosphere.



"Suppose a spectator placed at A, any point of the earth's surface K A k; and let L l, M m, N n, &c., represent successive strata of decreasing density, into which we may conceive the atmosphere to be divided, and which are spherical surfaces concentric with K k, the earth's surface. Let O represent the object under observation, whether terrestrial or a heavenly body, within or without the utmost limit of the atmosphere; then, if the air were away, the spectator would see it in the direction of the straight line A O. But in reality, when the ray A O passes from a rarer into a denser stratum, suppose at d, it will by the laws of optics begin to bend *downwards*. But as it advances downwards, the strata continually increasing in density, it will continually undergo greater and greater refraction in the same direction; and thus, instead of pursuing the straight line O d A, it will describe a curve O d c b a, continually more and more concave downwards, and will reach the earth not at A, but a certain point a nearer to O. This ray, conse-

quently, will not reach the observer's eye. The ray by which he will see the object O is, therefore, not O d A, but another ray, which, had there been no atmosphere, would have reached the earth at K, a point behind the observer; but which, being bent by the air into the curve O D C B A, actually arrives at A. Hence the object O will be seen, not in the direction O A, but in that of A o, a tangent to the curve O D C B A at A. But because the curve described by the refracted ray is concave downwards, the tangent A o will lie above A O, the unrefracted ray; consequently, the object O will appear more elevated above the horizon H R, than it would appear were there no such atmosphere. Since, however, the disposition of the strata is the same, or assumed as being the same, in all directions around A, the visual ray will not be made to deviate laterally, but will remain constantly in the same vertical plane O A E passing through the eye, the object, and the earth's centre*."

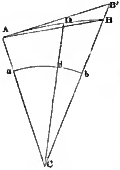
Exceptions to this rule have been observed, and lateral deflection has been the consequence of a supposed subversion of equilibrium in the same concentric ring. Under certain states of the atmosphere denser strata have also been supposed to be temporarily incumbent on rarer strata, the curve or path of the refracted ray becoming in such a case convex downwards, whereby a double curvature is produced, the effects of which there are as yet no means of estimating, and consequently correcting:—such cases fortunately are of rare occurrence.

We now proceed to the investigation of a formula for measuring this refraction, supposing it to occur only in a vertical direction, and thus tending to raise the apparent position of the object.

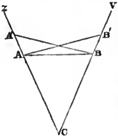
Let C be the centre of the earth, and $a\ d\ b$ its surface; if from a station A, a distant object B be observed, the visual ray from B will describe the curve B D A, and the object

* HERSCHEL'S *Astronomy*, p. 27.

will appear situated at B′, in the direction of the tangent to the curve at A. The angle B A B′ therefore is the measure of the displacement caused by refraction.



The nature of the curve B D A is unknown; but as in all geodesical operations the distance A B is always comparatively small, the curve B D A may be assumed circular, as being an arc of the osculating circle to the curve. Under this hypothesis, the angle B A B′ is equal to half the arc A B. (Euc. III. 20 and 32.) With an object D, the refraction would be measured by half the arc A D, hence the refraction is proportional to the arcs A D, A B. But the arcs A B, A D may be considered as proportional to the arcs *a d*, *a b* on the earth's surface; hence the amount of atmospheric refraction varies as the angle formed by vertical lines drawn from the extremities of the curve of refraction; or making R = refraction, and C = angle at earth's centre, then $R = n\,C$, n being a coefficient deduced from experiments, and which remains constant in the same state of the atmosphere. The following is the method adopted to obtain the value of this coefficient, which varies according to the elevation of the object above the horizon of the observer.



Let C, as before, be the centre of the earth, and A and B two stations. The station B, observed from A, will be seen in B′ owing to the effect of refraction; and the station A, observed from B, will appear in A′ for the same reason. The angles between each object and the zenith, (*i. e.*, the sum of

the angle of depression and 90°, or the difference between the angle of elevation and 90°,) will, when observed, be diminished by the measure of the angle of refraction, *i. e.*, the zenithal angle at A will be Z A B′, and at B, V B A′.

The exterior angle of a triangle being equal to the two interior and opposite, we have

$$Z\,A\,B = A\,C\,B + A\,B\,C,$$

$$V\,B\,A = A\,C\,B + B\,A\,C, \text{ and}$$

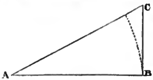
$$Z\,A\,B + V\,B\,A = 2\,A\,C\,B + A\,B\,C + B\,A\,C = 180° + A\,C\,B.$$

From this equation we find that the sum of the true zenithal angles of the two stations is equal to two right angles + the contained arc; hence the excess of this quantity above the *observed* zenithal distances + the contained arc, will give the measure of the sum of aberration due to refraction in both observations. If the observations at A and B have been made precisely at the same moment, and when the state of the atmosphere therefore would have had the same effect on both observations, the amount of error divided by 2 will give with precision the amount of correction to be made at each angle. It is extremely difficult, however, with great distances, (those in which the correction is most wanted,) to make simultaneous and reciprocal observations of this kind; but a series of observations should be taken at each station under the most favonrable circumstances, about noon of a cloudy calm day, when in our climate the tremulous motion in the air is commonly the least; the mean of the results is fairly assumed as the value of the coefficient n in the equation $R = n\,C$.

Let D and d represent the difference obtained by subtracting 90 from each zenithal distance; then without refraction, we should have $D + d = A\,C\,B$: and if the sum of the refractions $= 2\,R$, then $2\,R = A\,C\,B - (D + d)$, care being taken to give the proper signs to D and d, which become negative when the zenithal distance is less than 90°:—the mean refraction at each station is therefore

$$R = \frac{A\,C\,B - (D + d)}{2}$$

In thus arriving at the value of the mean refraction which is to be used as a correction, care must be taken to reduce the elevations or depressions to the place of the axis of the telescope, in case the ground or a signal have been observed. This is effected by adding or subtracting, (as the case may require,) the angle subtended, at the place of observation, by the vertical height between the object whose elevation or depression was observed, and the axis of



the telescope when at that station. Thus, let B C be the height of the telescope C, above the ground B observed, and A B the distance between the two stations,

$$\text{then } A\,B : B\,C :: \text{Rad.} : \text{tang. } A,$$

$$\text{tang. } A = \frac{B\,C\,.\,\text{Rad.}}{A\,B}$$

At the distance of 206,265 feet, 1 foot subtends 1″. At one mile, 1 foot subtends 39·06″ nearly.

It is customary to express the amount of refraction in terms of the distance between the stations in degrees or parts of a degree. This expression for the distance, which is then called the "contained arc," is obtained by the following proportion.

$$365110 \text{ feet} : \left\{ \begin{matrix} \text{distance between} \\ \text{the stations} \end{matrix} \right\} :: 1^\circ : \text{contained arc},$$

the length of 1° at the earth's circumference being at a mean valuation equal 365,110 feet, or 69·15 miles.

When the angle of elevation exceeds 8° or 10°, as in astronomical observations, the amount of refraction has been ascertained with precision by the comparison of the results of a great number of observations made as follows. A circumpolar star which passes the zenith, and another which grazes the horizon, are followed with an altitude and azimuth circle, (an instrument constructed on the same principles and with the same movements as the theodolite),

through their whole diurnal course; and the exact apparent form of their diurnal orbits, or the ovals into which they are distorted by refraction, are traced. Their deviation from circles, being at every moment given by the reading of the vertical arc, gives the measure of the refraction due to all degrees of elevation; which is found to decrease rapidly, from the horizon where it is greatest, to the zenith where it becomes nothing. Accurate tables of the mean astronomical refractions are prepared, by means of which the angles of elevation of celestial bodies are to be corrected.

When the object observed is nearer to the horizon than 8° or 10°, the refraction, then termed terrestrial refraction, has been found to vary in a very irregular manner, changing materially with all changes in the state of the atmosphere. Different values for this co-efficient have therefore been adopted by different observers. General Roy, in the operations of the Trigonometrical Survey, assumed it at $\frac{1}{15}$th or $\frac{1}{12}$th of the contained arc in cases where it had not been ascertained by actual observation; but in examining the correction for refraction obtained from actual observations in that survey, we see it varying from $\frac{1}{4}$th to $\frac{1}{34}$th of the contained arc; the greater number of these corrections, however, oscillating between $\frac{1}{10}$th and $\frac{1}{15}$th. It is evident, therefore, that, for terrestrial refraction, it is impossible to generalize a formula in the present state of knowledge; and extreme cases of extraordinary refraction have been mentioned which no previous calculations could have prepared the observer to guard against. When tracing out the base on Hounslow Heath, General Roy had directed the telescope fixed at King's Arbour towards Hampton Poor House, "at which end of the base a flag-staff had been erected; this, for a long time, he endeavoured in vain to discover, till at last, very unexpectedly, it suddenly started up into view, and so high it seemed to be lifted, that the surface of the ground where it stood became visible. This will appear the more extraordinary, when it is considered

that a right line drawn from the eye of the observer at King's Arbour to the other end of the base, would pass 8 or 9 feet below the surface of the intermediate ground."

"On the same base line, 30 pickets had been driven 100 feet from each other, so that their heads appeared through the telescope to be in a right line: this was done in the afternoon. The following morning proved uncommonly dewy, and the sun shone bright; when having occasion to replace the telescope, it was remarked that the heads of the pickets exhibited a curve concave upwards: in the afternoon, when the ebullition in the air subsided, the curve appearance was lost *."

Example 1 †. The distance between Maker Heights and Kit Hill Stations, in the eastern part of Cornwall, was computed at 67822 feet.

On Maker Heights, the ground at Kit Hill, elev. 29′ 45″
At Kit Hill, the ground at Maker Heights, depr. 37 38
Required the mean refraction at each Station,

$$\text{or } R = \frac{C - (D + d)}{2}.$$

To find C, the contained arc in degrees, we have the proportion,

$$365110 \text{ ft.} : 67822 \text{ ft.} :: 1^\circ : C = 11' \, 8{\cdot}7''$$

$$\left.\begin{array}{l} D = -\ 29'\ 45'' \\ d = +\ 37\ \ 38 \end{array}\right\} \text{as observed} \left\{ \begin{array}{l} = -\ 29\ \ 39 \\ = +\ 37\ \ 44, \end{array}\right.$$

a change of 6″ being made in the observed angles, owing to an error in the parallelism of the spirit-level, as explained in the next Example;

$$R = \frac{11' \, 8{\cdot}7'' + 29' \, 39'' - 37' \, 44''}{2} = 1' \, 32'' \text{ nearly,}$$

* *Trig. Survey*, vol. i., p 175.

† *Trig. Survey*, 1795.

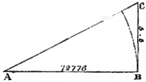
and dividing the contained arc $11' \, 9''$ by $1' \, 32''$, we obtain, as the value of the mean refraction, $\frac{1}{7}$th of the contained arc.

Example 2. In this example $6''$ have been deducted from the observed angle of elevation, and added to the angle of depression, on account of an error in the parallelism of the line of collimation of the telescope and the spirit-level.

Distance from Little Haldon to Furland 72776 feet, $= 11' \, 57{\cdot}6''$.

The height of the theodolite was 5·5 feet above the ground, then because

A B : B C :: Rad. : tang. A,



Log. 72776 . .	4·861988
: log. 5·5 . . .	0·740363
:: Rad. . . .	10·000000
: tang. 16 . .	5·878375

Therefore $16''$ are to be added to the angle of elevation, and subtracted from the angle of depression.

At Furland, ground at Little Haldon . . . } elev. $5' \, 21'' + 16'' = 5' \, 37''$

At Haldon, ground at Furland, depr. $16 \;\; 12 - 16 = 15 \;\; 56$

The sum of the zenithal angles, if there were no refraction, would be .... } $= 180° + C = 180° \, 11' \, 57{\cdot}6''$

Observed zenithal angles . } $= 89° \, 54' \, 23'' + 90° \, 15' \, 56'' = 180 \;\; \dot{10} \;\; 19$

Diff. $2 \,)\, 1 \;\; 38{\cdot}6$

Mean R $= \quad 49{\cdot}3$

$= \frac{1}{15}$th of the contained arc nearly.

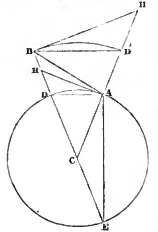
Example 3. The distance between Lansallos and Carraton Hill Stations, Cornwall, was computed at 68929·5 feet.

At Lansallos, the ground at Carraton Hill, elev. 30′ 18″
On Carraton Hill, the ground at Lansallos, depr. 41 18

Required the mean refraction, assuming that no corrections are required for parallax of the instrument, or error in the line of collimation.—*Answer*, $\frac{1}{11}$st of contained arc.

Levelling with the Theodolite by means of Trigonometrical Calculation.

Vertical angles in the mensuration of heights with the theodolite being estimated from the direction of the spirit-level or plummet, they are consequently measured by reference to a tangent to the surface of the earth at the point of observation.



Let the points A and B represent two remote objects, and C the centre of the earth. With the radius C A describe a circle, draw A H, tangent to the circle at A, and C B, cutting the circumference in D and E, and join E A and A D. Then B H will be the apparent, and B D the true, elevation of B as compared with A. Through B, with the radius C B, describe the arc B D′, and draw B H′ tangent to the arc B D′ at B. Then if B is observed from A, the angle of elevation is too small by the angle D A H; if A, on the contrary, is observed from B, A H′ will be the apparent, and A D′ the true, depression of A below B, the

angle of depression being too large by the angle D′ B H′. The value for H D, the difference between the true and apparent elevation, has been given (page 123) in terms of the distance A H; it may be given in terms of the angle at C, being $= \frac{1}{2}$ C, for D A H = angle A E D in the alternate segment of the circle $= \frac{1}{2}$ A C D (Euc. III. 20).

Hence the true vertical angle at any station will be found, by adding to the angle observed with the theodolite when it is an angle of elevation, and when it is an angle of depression deducting from it, half the measure of the contained or intercepted arc.

This measure depending on the curvature of the earth, which is neither uniform nor regular, should (mathematically speaking) be deduced, for each particular place, from the length of the corresponding degree of latitude. Such nicety, however, is very seldom required. It will be sufficiently accurate in practice to assume the mean quantities, and to consider the earth as a globe. Assuming its mean diameter at 7916 miles, the arc of a minute on the meridian would be equal to 6085 feet nearly, and the correction to be added to the observed vertical angle will amount to one second nearly for every 101 feet contained in the intervening distance.

The vertical angle being hence obtained, and the horizontal distance between the two stations computed from previous data of the triangulation, the height B D, or the difference of elevation between A and B, is obtained by a simple computation from the triangle B A D, in which the side A D is given, the angle B A D = observed angle $+ \frac{1}{2}$ A C D, and the angle D B A $= 90° -$ (observed angle $+ \frac{1}{2}$ A C D); for A D B may be considered a right angle.

Example. Taking the distance between Maker Heights and Kit Hill as given, (page 130,) as also the angles of elevation and depression corrected from the errors due to

refraction; required the height of Kit Hill, Maker Heights having been found by levelling with the spirit-level to be 402 feet above the sea at low water.

The angle of elevation at Maker Heights, corrected from the effect of refraction, is equal to . . . 28′ 7″

to which, adding half the contained arc, or . . $\frac{11' \; 9''}{2}$

the true angle of elevation becomes . . . . 33′ 41·5″; then because,

Rad. : tan. angle of elev. :: horiz. dist. : diff. of elevation, we have,

Log. rad.	10·0000000
: log. tan. 33′ 41″ .	7·9911551
:: log. 67822 feet .	4·8313706
: log. 664·6 . . .	2·8225257

Maker Heights elevated above the sea	402 feet,
To which add difference of elevation .	664·6 „
Kit Hill elevated above the sea . .	1066·6 „

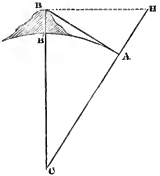
It is to be observed, that unless reciprocal angles of elevation and depression have been taken in the same state of the atmosphere, at each station, the levels obtained cannot be securely depended upon, owing to the constantly varying condition of the atmosphere, and the consequent difficulty of ascertaining the true coefficient of correction. As an instance of the great discrepancies that refraction has been known to cause, we may notice* the height of St. Ann's Hill, as obtained from observations taken by General Roy in 1787, at the station near Hampton Poorhouse, and that deduced for the same point from observations made at the same station in 1792, when, the axis of the instrument being at the same height above the ground,

* *Trig. Survey*, vol. i., p. 172.

the angle of elevation in the first case was 17′ 39″, and in the second 8′ 11″. The height deduced from the first observation was 321 feet, and that deduced from the second 240 feet.

As to the relative heights obtained by the observation of the reciprocal angles, they cannot, unless the reciprocal observations have been made exactly at the same moment, be depended upon as approaching nearer the truth than about 10 feet*, with triangles whose sides are from 7 to 10 miles, and so on in proportion. The best time for making such observations is on a cloudy day, when the tremulous motion in the air is commonly the least.

When the horizon of the sea is visible from an elevated station, its altitude above the level of the sea can be ascertained by a simple observation with tolerable accuracy, as follows†.



Let B be the station of the observer, from whence the line B A is drawn tangent to the surface of the sea. The angle of depression H B A is equal to the observed angle, plus the correction for refraction which must be assumed at a mean value.

In the right-angled triangle A B C, the elevation of the point B, or

$$B\,B' = C\,B - A\,C, \text{ and } C\,B = \frac{A\,C}{\cos. C}, \text{ therefore,}$$

$$B\,B' = \frac{A\,C}{\cos. C} - A\,C = \frac{A\,C - A\,C \cos. C}{\cos. C}$$

$$= A\,C.\,\frac{(1 - \cos. C)}{\cos. C} = A\,C.\,\frac{(1 - \cos. C)\sin. C}{\sin. C \cos. C};$$

* *Trig. Survey*, vol. i., p. 173.

† PUISSANT'S *Géodesie*, vol. i., p. 355.

but because (trig.)

$$\frac{\sin. C}{\cos. C} = \tan. C, \text{ and } \frac{1 - \cos. C}{\sin. C} = \tan. \tfrac{1}{2} C,$$

we have by substitution,

$$B\,B' = A\,C \tan. \tfrac{1}{2} C \tan. C.$$

Whatever may be the height of the station of observation B on the earth's surface, the angle of depression, which is equal to the angle at C, will always be very small; in such a condition of the triangle we may assume (trig.)

$\tan. \frac{1}{2} C = \frac{1}{2} \tan. C$; then by substitution

$$B\,B' = \frac{A\,C}{2} \tan.^2 C.$$

Making the *observed* angle of depression $= D$, and assuming the correction for refraction in this climate at the mean value of $\frac{1}{10}$th of the contained arc,

$C = D + \frac{1}{10} C$, or $D = C - \frac{1}{10} C = C\,(1 - \frac{1}{10})$, whence

$$C = \frac{D}{(1 - \frac{1}{10})} = \frac{1}{(1 - \frac{1}{10})} D. \quad \text{By substitution,}$$

$$B\,B' = \tfrac{1}{2} A\,C \tan.^2 \left(\frac{1}{1 - \frac{1}{10}} \cdot D\right),$$ and because the angle D is very small,

$$B\,B' = \tfrac{1}{2} A\,C \frac{1}{(1 - \frac{1}{10})^2} \tan.^2 D$$

$B\,B' = \frac{1}{2} A\,C\,(1 + \frac{1}{9})^2 \tan.^2 D$, from which the required altitude $B\,B'$ is obtained nearly.

Example. Required the height of a station from which the horizon of the sea was seen depressed 22′ 36″, taking the correction for refraction at $\frac{1}{10}$th of the contained arc, and the radius of the earth = 20,888,000 feet.

Log. $\frac{1}{2}$	$\bar{1}$·6989700
Log. 20,888,000 feet	7·3198969
Log. $(1 + \frac{1}{9})^2$............	0·0915140
Log. tan. ^{2} (22′ 36″)	5·6356816
Log. 557·3 feet	2·7460625

Height of station above the level of the sea = 557 feet.

Throughout the preceding remarks no account has been taken of the spheroidal shape of the earth, and in all levelling operations of this character, even those connected with the most delicate geodesical operations, it may be neglected; for the effects of refraction and the errors always inseparable from angular observations are more considerable than the errors produced in geodesical operations by neglecting the extremely minute influence of the shape of the earth as differing from that of a sphere.

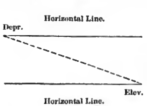
Having now explained the method of levelling with the theodolite, and described the corrections requisite in the operation, it remains only to observe that in the progress of the triangulation previously described, the angles of elevation or depression are taken to each station, for the purpose of obtaining the relative elevation of each, and their absolute altitude above the level of the sea. These angles of inclination are entered in an additional column in the field book; or, if the work be extensive, it is better to enter these angles in a separate field levelling book, in the following form.

From	To	Horizontal Reading or Index Error.	Apparent Elevation or Depression.	Reduction of Hypot. to Horiz. Base per 100 feet.	Remarks.
		′ ″	° ′ ″		
A	B	0 20	Depr. 3 0 0	0·14	
A	C	0 20	Elev. 6 30 0	0·65	

In our description of the adjustments of the theodolite it was remarked, that when the adjustments of the optical axis, parallelism of the level, and horizontal position of the circular limb were perfect, the index of the vertical arc should point to zero; but as some alterations unavoidably take place in these delicate adjustments by the carriage of the instrument, the third column is designed to receive the entry of the index error. No entry is, of course, made

here, if the method of compensating for this error described in page 51 be adopted. The fourth column contains the apparent elevation or depression of the object; in the fifth are entered the number of feet to be subtracted per 100, as shown on the vertical arc; and the sixth is left for remarks. Among these remarks, a few horizontal angles to surrounding objects should occasionally be entered.

As before observed, if from any station the telescope be directed towards the ground, or the top of a signal at the next station, a correction is required in the observed vertical angle. If, for example, the ground be the point observed, the correction is additive if the vertical angle be one of elevation, and negative if the observed angle be one of depression. To avoid the necessity for this correction, the instrument should be set up (as nearly as possible) at a constant height above the ground, and the staff used for the observations should have a cross bar or vane fixed at the same height above the ground as the axis of the telescope, which bar or vane is to be bisected by the cross wires in observing. This precaution cannot always be used in the primary triangulation, and is, of course, inadmissible when the observed objects are permanent structures, such as church steeples, towers, &c.; but it should be universally adopted when levelling for the interior detail of a survey. In this latter case, the reciprocal angles of elevation and depression should be taken in order to ensure accuracy, and when the distances are so short that the effects of curvature and refraction are not sensible, the reciprocal angles, if observed correctly, ought to be equal to one another, the horizontal lines at each station being to our senses parallel.



In cases where the distances are short, and the relative altitudes are not required, the reduction of the lines to the

horizontal plane, previously to their being used in plotting, may be made by reference to the column for the reduction of the hypotenuse to the horizontal base, in the levelling field-book. But when the distances are long, or the relative altitudes required, logarithmic computations should be used. In an extensive survey, time will be saved, and errors guarded against in this operation, if the entries for the calculation are made according to the following form, thus described in COLONEL COLBY's *Instructions for the Interior Survey of Ireland.*

Plan and Plot.	Measured distances.	Elevation or depression.	Calculations of reductions to the horizon. — Hyp. × cos. of angle of inclination.	Horizontal distances.	Calculation of vertical distances. — Hyp. × sin. of angle of inclination.	Relative altitude.	Altitude above low-water mark.	Remarks.
							A 21·50	Obtained by levelling.
B	A 1942 B	elev. 2° 29′ 30″	9·9996351 3·2882192	1940·3	8·6382280 3·2882492	84·43	105·93	
			3·2878813		1·9264772			
	B 2156 C	depr. 1 21 20	9·9998801 3·3336488	2155·4	8·3739542 3·3336488	51·00	54·93	
			3·3335292		1·7076030			
	C 859 D	elev. 4 27 0	9·9986888 2·9339932	856·4	8·8898007 2·9339932	66·65	121·58	
			2·9326820		1·8237939			

"In the first column of this register, the designations of the plans and plots in which the points or lines are contained are entered. The second column shows the measured length in feet of the station line, which length is to be written between the letters marking its extremities, thus, A 1942 B. The third column shows the mean elevation or

depression of the second object deduced from the reciprocal angles in the levelling field-book, after applying the corrections indicated in the third column of that book, and those for curvature and refraction when very long distances render their effect sensible. The fourth column contains the logarithmic cosine of the angle in the preceding column, and the logarithm of the distance; the natural number answering to the sum of these logarithms is entered in the fifth column. The sixth column contains the logarithmic sine of the angle, and the logarithm of the distance; the number answering to the sum of these two logarithms is entered in the seventh column. The eighth column contains absolute altitudes above the low-water mark. The altitudes in this column are to be proved by always commencing at some point whose altitude is known, either from the Trigonometrical Survey or by levelling with the spirit-level, and proceeding in a regular series of additious or subtractions to some other point of which the altitude is also known in like manner. In connexion with these levelling operations, observations should be made for the purpose of ascertaining the heights of the rise and fall of the tide, both at springs and neaps, at various places on the coast, &c., the altitude above low-water (spring tides) of some conspicuous part of each of the points which has been trigonometrically determined; and of a sufficient uumber of other points, found by levelling, &c., to prevent the accumulation of error in the altitudes given in the register.

"The survey thus performed will furnish a great number of accurate heights, at short distances from one another, over the district surveyed; it will be easy to render this part of the work complete, and subservient to future local improvements, without devoting much additional time to this object. Not only the heights of hills, but also those of the lowest parts of the necks which connect them should be given; also the heights and depths of lakes, and the

altitudes of rivers and streams in various parts of their courses. As churches are usually very permanent objects, the heights of the ground on which their towers or belfrys are erected should be given as points for future reference: and a knowledgo of the altitudes of mines and mineral deposits, and of manufactories, towns, and villages, will tend to facilitate internal improvements. The heights of canals should bo given at all the locks, and the heights of the summit levels of roads; and also, when it can conveniently be done, the height over which a new canal or road must unavoidably pass to connect a valuablo mineral deposit, or principal market or manufactory, with some adjacent harbour, navigable river, or existing canal."

CHAPTER V.

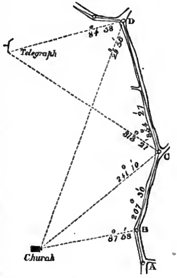
INTERIOR DETAIL OF A TRIGONOMETRICAL SURVEY.

The triangulation for a survey being accomplished, the filling in of the interior detail presents no difficulty. The larger triangles being subdivided into others of a smaller size, the sides of these are measured with the chain, and the field-book is kept according to the form given in the first Chapter. While measuring the sides of the triangles or station lines, the surveyor takes the angles of elevation and depression, both for the purpose of reducing the inclined lines more correctly to the horizontal plane, as also to obtain as many altitudes as possible over the surface of the district surveyed. With this measurement by the chain, the surveyor can enter into the most minute detail, and give the true form of all the fields and other enclosures. His object, however, may not always be to make detailed property plans, but simply to lay down the roads, rivers, boundaries of woods, and other great lines of artificial or natural demarcation. In this case, the survey of the roads, rivers, woods, &c., is made with the chain and theodolite, according to a process to which the term "traversing" is applied. In describing the process, we shall take as an example the survey of a road, to which it is most frequently applied.

Traversing.

At the starting point, distinguished by a letter, (say A,) a staff, with a cross bar fixed at the usual height of the axis of the theodolite above the ground, is erected; and from A a straight line is measured in the direction in which the road is to be surveyed to a point B, selected at or near the first turn or bend in the road, offsets being taken to the

right and left, as may be required, to determine the width and boundary of the road: the measurements are entered as usual in the field-book. At B the instrument is adjusted and set to zero, the telescope having first been directed to the staff at A: the upper plate being then unclamped, the readings of one or more conspicuous objects in the neighbourhood are taken; and lastly, the telescope being directed upon a staff fixed by an assistant at the forward station C, chosen at the next bend in the road, the upper plate is



firmly clamped, and the *forward* angle read off. Proceeding from B to C, the line B C is measured with accompanying offsets as before, and the theodolite adjusted over the station C. The ranging staff, brought forward from A and fixed at B, serves as the object on which the telescope is to be directed for the *back* angle, by means of the movement of the *lower* limb, the upper plate still remaining clamped to the last forward reading. The staff at B being properly intersected by the cross wires, the lower plate is then firmly clamped, and the upper plate having been unclamped, the same conspicuous objects are intersected as before, and the telescope is afterwards directed to the next forward station D,—when the same operation is repeated, to be afterwards continued throughont the entire length of the road to be traversed.

It will be observed that, by leaving the upper plate clamped for the *back* reading at the same angle as the pre-

ceding *forward* reading, the readings of the horizontal limb, for the angles forward, indicate the direction of each station with reference to the first line on which the telescope was set, which may therefore be called the first meridian. This method of reading saves, in the subsequent plotting, the trouble of changing the position of the protractor at every angle.

The angles read at each station may be entered, at the corresponding station in the field-book, at the end of the measurements relating to the station line immediately preceding; but if the road or other line to be surveyed be a long one, it is better to register the angles in another part of the field-book, and according to the following form, which provides also for the entries of the angles of elevation and depression.

Hypotenuse.	Measured distances in feet.	Elevations or depressions.	Reading of conspicuous objects or stations.	Readings of horizontal limb.		Remarks.
				Back	Forward.	
A (1) B	300	depr. 3° 30′	Church ....87° 58′ 20″	0° 0′ 0″	207° 30′ 0″	
B (2) C	723	elev. 3 30 elev. 7 21	Church .. 241 10 0 Telegraph 313 21 0	207 30 0	354 27 20	
C (3) D	1218	depr. 7 21 elev. 2 6	Church ....29 55 40 Telegraph ..84 58 10	354 27 20	106 52 0	

In the above example, from which the diagram in the previous page has been plotted, the line A B is the first meridian to which all the angles are referred; but during the progress of the work other lines are selected as meridians, in order to facilitate the plotting. To constitute any one of the lines a meridian line, it is only necessary to fix the vernier at zero for the back angle, instead of retaining the preceding forward angle.

Pickets are generally driven at each station, to mark the precise spot, should it be necessary to refer to it again; and at the close of the day's work, if the survey is incomplete, angles should in addition be taken to several fixed points near the last station, in order that it may be identified with ease when the work is resumed.

The reason for taking angles to surrounding conspicuous objects, (if they be not so distant as to make the angles of intersection very acute, and thereby form ill-conditioned triangles,) is that they may serve as a check on the work; for the several bearings on the same object should, in the plotting, meet at a common point of intersection. However, from the dependence of each new station line for its direction upon those that precede it, errors will, except under favourable circumstances, be introduced into the work. The different parts of a traverse should therefore be referred to certain accurately fixed points, and any errors thus discovered, may, if small, be corrected and allowed for in the plotting.

The surveyor, in plotting, marks off around the first station the bearings of the angles referred to the first meridian; and, taking from the field-book, in succession, the length of each station line connected with that first meridian, he transfers the direction from the central point by means of a parallel ruler. The same operation is repeated around each station at which a new meridian was assumed. The station lines, when checked and found correct as to length and direction, are marked in faint red lines, and the road itself is plotted from the offsets given in the field-book.

Highly-finished circular protractors are made in metal with verniers attached, capable of reading to minutes and fractions of minutes; but these protractors, being steadied by pins, are liable to be altered in position by the movement given to the arms bearing the verniers. For general use, circular pasteboard protractors, with the interior circle cut out, will be found more convenient;—these can be

made by describing a circle with a radius of 5 inches, and dividing the circumference to one-half or one-fourth of a degree, by measuring from an accurately-divided diagonal scale the lengths of the chords as given in the table of natural sines, and applied in the manner explained in page 121. Quantities less than one-half or one-fourth of a degree may be estimated by the eye.

In road or other traversing, it would scarcely be necessary to read to seconds, if the direction of the lines were to be plotted solely by reference to the protractor. But independently of the fact, that, after a little practice, no more time is required to read the seconds than the single minutes, it may be observed here, that when great accuracy is sought, the direction of the lines is calculated by the resolution of right-angled triangles, in which the angle registered forms one of the data. It is especially when traversing for the surveying of mines, that calculation may be thus resorted to with advantage; we have therefore reserved for the chapter on Mining Surveys the necessary explanations on this subject.

Town Surveys. Surveys of towns or villages are made in the same manner by *traversing;* but they may be much expedited by taking angles from spires or commanding heights to remarkable points presented by the buildings, such as chimneys, remarkable roofs, projections, &c., noting them in the field-book, with slight sketches of each corresponding object observed, in order that it may be easily recognised. Such intersections taken from three points, fixed in position, will give the site of the principal buildings; and when these are laid down, traverses run in the intermediate streets and roads will give the details with accuracy, the angular intersections serving as a check on the directions of the traverse lines.

Surveys for Railways, or projected Lines of Communication. After what has been said of surveying in general,

a few words will suffice to explain the mode of proceeding with surveys for projected lines of communication. These surveys being, from their very character, confined to a narrow strip of country, (frequently not extending more than a few hundred yards in width,) preclude the advantageous application of a system of triangulation. It may be observed, however, that such lines of communication as demand the construction of accurate plans of the properties they pass through, are projected only in districts in which civilization has made considerable progress, and in which, in most instances, a trigonometrical survey of some kind will be found to have been performed at some preceding period. Access may always be had in those cases to documents, giving descriptions of the various stations whereby they may be recognised on the ground, and furnishing the data necessary for calculating their respective distances. In England, for example, as well as in most continental states, the lengths of the sides of the primary and secondary triangles, with all the angles observed, whether to trigonometrical stations or permanent conspicuous objects, are published:—a surveyor, therefore, who undertakes a survey for a railway, or any other work embracing a considerable extent of country, would do well to avail himself of the data furnished by those publications, using them as checks on the results of his own less careful and more hasty labours.

As regards the mode of proceeding with the survey, for a railway for example, the system to be followed will resemble that described for road traversing, with this exception, that the station lines, instead of being a few hundred or thousand feet in length, may be made to extend from five to ten miles, according to the general direction of the proposed work and the nature of the country. These great station lines are connected with each other by repeated and most careful observations of the angles of their intersections, and also by angles taken to the trigonometrical points we

have before referred to, or to other conspicuous objects. The station lines, thus determined in direction, and measured with due care, serve as great bases, on each side of which small offset-triangles are measured to embrace all the detail required; the field-work being conducted, and the field-book kept, according to the method described in the first chapter.

The completion of surveys, in which many principal points are determined, and of which a portion of the outline has been measured and plotted, may be expedited by taking the plan itself to the ground, and measuring with the box-sextant the angles between fixed points, or taking their bearings with the meridian by the prismatic compass; the directions of the lines and the measured distances being at once protracted on the plan, and no field-book therefore being kept. A protractor scale is required to set off the angles or bearings; one of its edges being divided as a plotting scale suited to the scale on which the plan of the survey is to be drawn. When plans are thus required to be taken to the field, they should be traced on bank post paper, which admits of being folded without injury, so as to expose any portion of the plan, and only that portion which may be required at one time. In order to enable the surveyor to draw or plot more neatly, the paper may be folded over a rectangular piece of board, fitting on a field sketch-book or portfolio, and retained in place by leather bands at each corner.

It is not unfrequently of advantage (to the military surveyor especially) to be able to construct a plan, or to fill in the interior detail between fixed stations, with tolerable accuracy, without instruments. By pacing, distances may be measured; bearings or directions taken with the aid of of a straight walking stick; and the distances plotted at

once on the field sketch-book by the aid of a plotting scale: —the intention of such reconnoissances, and their chief recommendation, being, that the plan may be constructed at the same time that the surveyor walks over the ground. The greatest difficulty consists in setting out correctly on the field-sketch the first few objects; their relative distances should therefore be paced, and their bearings taken with care. They may serve, when determined, in giving the position of other points or objects, by the intersections of two or more lines passing through these objects fixed in position. The expeditious method of measuring distances approximatively by pacing, is not so liable to error as might be supposed, if due precaution has been taken to ascertain the length or value in feet or yards of a given number of steps made at the *common walking pace.* An attempt to make each step equal to one yard would produce greater inaccuracies, because of the difficulty of preserving for any length of time the steps of an unusual length. It is better to ascertain their length by actual trial, by pacing several times a known measured distance at the usual rate and pace. A very general measure of the length of steps is 24 steps to a chain, or to 22 yards.

It may not be out of place here to describe a method of measuring inaccessible distances, especially the width across a ford or a river, which experience, unassisted certainly by geometrical knowledge, has taught the peasantry of some countries to devise*. They place the top of a stick resting on the foot or knee, and kept as steady as possible in a line with the eye and the remote inaccessible object whose distance is required: they then turn the head and body round, and watching the point where the ray passing from the eye to the top of the stick intersects the ground on an accessible spot on the same level, they pace to the point of inter-

* MALORTIE's *Topography.*

section thus found, and thereby obtain an approximation to the distance required.

Independently of pacing, practice in sketching outline plans trains the eye to judge short distances with tolerable accuracy. A knowledge of the laws of perspective, both linear and aërial, is also of some assistance; for as perspective teaches to transform actual into apparent forms, so it may teach to deduce actual from apparent forms.

CHAPTER VI.

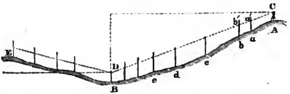
LEVELLING FOR SECTIONS.

Levelling with the Theodolite.

When the theodolite is used in levelling for sections along a continued straight line, much time would be lost by placing the instrument at every change of level which it is desired to mark on the section. The following method is adopted to obtain the section required.

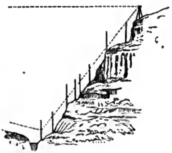
The section line having been ranged, and pickets driven at all great changes of inclination of the ground, the theodolite is set up at one extremity of the line, and the intersection of the cross wires made to bisect the vane on a staff erected at the site of the first picket, the vane being as nearly as possible of the same height as the axis of the theodolite. The angle of elevation or depression having been noted, a levelling staff with a sliding vane is taken by an assistant to each irregularity of the ground offering itself in succession between the observer and the second station, and the vane is raised or depressed on the staff according to signs made by the observer, until the centre of the vane is intersected by the cross wires. When the vane is thus fixed, the reading of the staff is noted by the assistant; this mode of levelling being only resorted to when the distances are too great to enable the observer himself to read the divisions on the staff. For example: Let A be the position of the theodolite at the first station, and B that of the staff fixed at the second station. Between A and B, the intermediate positions a, b, c, d, &c., for holding the levelling staff, are determined by the irregularities of the ground. The angle of depression to B is observed, and the vertical

arc being clamped in that position, the assistant places the levelling staff successively at a, b, c, &c., the centre of the



vane being brought into the line of sight C D, and the heights $a\,a'$, $b\,b'$, &c., noted. The instrument is afterwards brought to B, from whence the reciprocal angle of elevation to C is observed as a check on the work, and the telescope afterwards directed upon the staff fixed at the third station E in the line of section. The theodolite being clamped in that position, the same operation is repeated to note the irregularities between B and E. "In laying the section down upon paper, a horizontal line being drawn, the angles of elevation and depression can be protracted, and the distances laid down on the inclined lines as they were measured by the chain at the time of the observations being taken. The respective height of the vane of the staff being then laid off from these points in a vertical direction, will give the points a, b, c, &c., marking the outline of the ground. A more correct way, of course, is to calculate the difference of level between the stations, allowing in long distances for curvature and refraction." As regards this method of levelling for sections, it may be observed that in all cases it is inferior in point of accuracy to the levelling with the spirit-level, and that it seldom saves much in point of time. A serious objection to it, is the necessity for entrusting the reading of the staff to an assistant, and for making a previous careful inspection of the line to determine on the site of each station of the theodolite, in order that the staff, when held on the intervening irregularities, may be long enough to be intersected by the

line of sight. This mode of levelling for sections should not therefore be adopted, except in cases where the line of country, of which a section is required, is intersected by deep and precipitous ravines, to cross which much time is consumed when levelling with the spirit-level, owing to the difficulty in fixing the instrument in places suited to the proper reading of the staff, and owing also to the great number of readings required in a short horizontal distance. With the theodolite, on the contrary, it is sufficient to place the instrument at the top or bottom of the ravine, and take the differences of level as above described.



In thus levelling with the theodolite when the distances from station to station are long enough to make the effects of curvature and refraction sensible and of practical importance, they should be taken into account. It rarely happens, however, that such a correction is required, because, within such distances as are adopted in practice, the corrections due to these causes are more minute than the errors caused by the difference of elevation between the axis of the theodolite and that of the vane above the ground, and other disturbing causes. For example, the correction for curvature and refraction combined is,

at $\frac{1}{4}$ mile,	only	0·0357	foot
$\frac{1}{2}$	„	0·1430	„
$\frac{3}{4}$	„	0·3216	„
1	„	0·5717	„

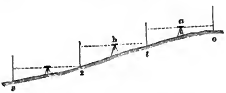
and as the vane is to be raised or depressed according to signs made by the observer at the instrument, a greater

distance than half a mile between station and station is but rarely adopted.

Levelling with the Spirit-Level.

The spirit-level, with its present improved construction, presents the most accurate means of obtaining a section along a continuous line, or of ascertaining the difference of level between isolated stations.

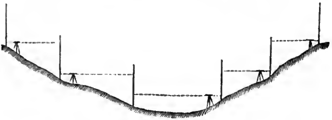
The spirit-level serves to trace a series of lines tangent to a great circle passing through the axis of the instrument, the centre of which circle is the centre of the earth; and if the instrument be placed *in the middle* of each of these straight lines successively, the difference of level between the extremities of each line will be obtained without any error arising from curvature or refraction.



The mode of proceeding is thus: The level is fixed at *a*, and adjusted by the parallel plate-screws; the difference of reading between the first or "back" station *o*, and the second or "forward" station 1, is registered in the field-book opposite the distance between the stations; the staff at the station 1 is kept unmoved, while the instrument is taken forward and fixed at *b*. From *b* the reading of the staff at 1, which then becomes a "back" station, is registered, as also the reading of the next "forward" station 2; from these data the difference of elevation between, not only the stations 1 and 2 is obtained, but also, by combination, the difference between the extremes 0 and 2 is given. The same process is continued for any required distance, giving the

elevation of each intermediate point, as well as the relative height of the extremes.

In common levelling operations, corrections for curvature and refraction may be neglected even when the instrument is not placed half-way between the staves, because the distance at which the staves can be read is so small as to render their effects inappreciable; at a distance of 500 feet, for instance, the correction due to both causes is only 0·00513 of a foot. Therefore, when the line of collimation is itself properly adjusted, the instrument need not necessarily be placed midway between the back and forward stations when a section of an inclined surface of ground is being taken; but its position may be so chosen that observations can be made each way, with the staves at a considerable distance from each other. When the ground is nearly level, it is better to fix the instrument midway between the staves; but when crossing a valley, the instrument, if properly adjusted, may be placed, for the sake of expedition, nearer the back stations in going down the inclination, and nearer the forward stations in rising on the opposite side: this arrangement avoids the



necessity of taking the sights inconveniently close to one another. The alternation also tends, by a compensation of errors, to correct in the final result the effect of any error that might exist in the line of collimation.

In levelling for a long section, bench marks or fixed stations that can be again found ought to be chosen at certain distances, rarely more than half a mile asunder, and

their elevation ascertained and registered as part of the section. Their use is to give greater facilities for checking the accuracy of the work as a whole, and for correcting errors that may have been made, without its being necessary to retrace the entire work a second time. Bench marks should therefore consist of permanent objects, so defined by a light sketch and description in the column for remarks, that they may be easily found again. Gate-posts, mile-stones, notches cut on stumps of trees, and similar points of reference, readily present themselves: for the prosecution of works in progress of execution, it is customary to drive short hard-wood piles in convenient places to serve as bench marks.

In levelling for a section, two chain-men are required by the surveyor to measure the distances, as also two staff-holders, who, as they place the staff at any particular distance along the chain, are to give the distance to the surveyor who stands by his level. Each staff-holder should be provided with an iron tripod, *i.e.*, a triangular piece of plate-iron, with its corners turned down to act as cramps whereby it may be steadily fixed in the ground; and with a hemi-

spherical projection on the middle of the upper surface on which the base of the staff is to be placed. The tripod, being firmly driven into the ground by pressure of the foot, serves as a fixed point on which the staff may, when upright, be turned round without the slightest change taking place in its elevation; whereas if no tripod, or other similar contrivance for obtaining a firm basis, be used, the staff, if resting on grass, clay, or gravel, &c., is liable to undergo a very sensible change in its elevation when turned round from one side to the other. Frequent repetitions of such alterations in its elevation would, in the course of a long section, introduce serious errors, which may be altogether avoided by the use of the tripod.

Some levelling staves have been constructed with a

tripod fastened to the base of the staff, by a pivot that admits of the staff being turned freely round. A decided objection to this construction is the liability which there is of the staff-holder unintentionally, especially in windy weather, or through carelessness, displacing the tripod by moving the staff: when, on the contrary, the tripod is separate from the staff, it is not liable, when once firmly fixed in the ground, to be moved, until taken up expressly after the observation has been made. Also in taking the level of bench marks, it is inconvenient to have the tripod fastened to the staff.

By means of a small plummet usually introduced in the side of the staff, the staff-holder is enabled to hold it upright in a vertical plane at right angles to a plane passing from the staff to the instrument. The vertical wires placed in the diaphragm of the telescope serve to detect any deviation in the staff from a true vertical position in the latter plane; and the surveyor, before registering the reading, takes care that the staff appears properly between these vertical wires, and parallel to them. That the staff, when observed, should be held in a truly vertical position is obvious; for if it be inclined, the intersection of the visual ray with the staff will give a number too great by the amount of difference between the leg of the right-angled triangle and the hypotenuse, which in the deviation from the vertical position forms one of the acute angles of the triangle.

The following is a good form for keeping the field levelling-book.

Rise.	Back Sight.	Fore Sight.	Fall.	Reduced heights.	Distance in feet.	Remarks.
				21·34		B.M. Top of key-
	3·86	4·88	1·02	20·32	0	stone, canal bridge.
1·53	6·43	4·90		21·85	100	
2·46	4·90	2·44		24·31	140	
·74	7·96	7·22		25·05	146	
	7·22	9·31	2·09	22·96	158	Fence.
2·04	9·29	7·25		25·00	167	
·93	7·25	6·32		25·93	200	
·79	6·32	5·53		26·72	240	
	4·53	4·71	·18	26·54	264	
1·46	4·71	3·25		28·00	300	
1·32	3·25	1·93		29·32	317	
	1·93	5·46	3·53	25·79	341	
1·11	8·24	7·13		26·90	352	Double fence.
	7·13	9·01	1·88	25·02	360	
2·51	9·01	6·50		27·53	365	
	6·50	7·91	1·41	26·12	400	
	7·91	8·65	·74	25·38	500	
	6·38	6·76	·38	25·00	600	
	6·76	6·76		25·00	700	
·96	6·76	5·80		25·96	800	
·28	5·80	5·52		26·24	880	
1·56	5·52	3·96		27·80	887	
	3·96	6·75	2·79	25·01	900	
1·62	7·69	6·07		26·63	903	Occupation road.
·04	6·07	6·03		26·67	921	
	6·03	7·58	1·55	25·12	930	
2·92	7·58	4·66		28·04	944	B.M. Plinth of bridge.
22·27	168·99	162·29	15·57			
15·57	162·29					21·34 + 6·70 = 28·04.
6·70	6·70					

The only entries registered in the field are those of the second, third, and sixth column, with those made under the head of remarks. In the second column are entered the back sights, in the third column the fore sights. And here it may be observed, that as each station becomes a forward and a back station alternately, the terms back and fore relate only to the respective position of two stations, and not to their position as being back or forward with reference to the position of the instrument or observer. When the level of any two points is taken, that point of station on which the first observation is made is a "back" station, that on which the second observation is made is a "forward" station, although both may be behind the instrument with reference to the direction in which the line is being levelled. There may be several sets of back and forward sights taken without the instrument itself being moved; in such a case, the reading of the fore sight of one set becomes the reading of the back sight of the next succeeding set, and is therefore repeated: an example of this is seen in the seventh line, and other subsequent entries in the form of field-book given in the adjoining page; 7·25, the fore sight of one line, is entered as the back sight in that which succeeds. This must be the case so long as the instrument remains unmoved, but when the instrument itself is carried forward, then the back sight ceases to be necessarily the same as the preceding fore sight, because the visual ray does not, after the displacement of the level, (except in some chance instances,) intersect the staff at the same elevation as it had done previous to its displacement. The distance corresponding to each fore sight is entered in the column for distances, in a line with the fore sight to which it refers.

The filling-in of the remaining columns of the field-book, or the *reducing* of the levels, is performed as follows:—the difference between each back and foro sight is taken; when the fore sight is less than the back sight, it is thereby

shown that the staff must have been in a higher position for the forward than for the back reading, a rise is therefore denoted, and the difference between the two readings is entered in the first column, under the head of *rise*. If, on the contrary, the fore sight be greater than the back sight, it is thereby shown that the staff must have been in a lower position for the forward reading than for the back reading; a fall is therefore denoted, and the difference between the two readings is entered in the fourth column, under the head of *fall*. The whole page being thus reduced, as a check on the arithmetical operations, the sum of the four first columns is taken, and if the additions and the subtractions be correct, the difference between the sums of the rises and falls will be the same as the difference between the sums of the back and fore sights. In the example we have given,

$$22{\cdot}27 - 15{\cdot}57 = 168{\cdot}99 - 162{\cdot}29 = 6{\cdot}70.$$

If the sum of the back sights exceed the sum of the fore sights, or the sum of the rises exceed the sum of the falls, a total rise is denoted: in our example a total rise of 6·70 is indicated; if the contrary holds, a total fall is denoted.

The fifth column, or that of reduced heights, remains to be filled. For that purpose, an arbitrary elevation above a base, called a datum line, is assumed as that of the first or starting point, the elevation of each succeeding point being obtained by adding or subtracting the corresponding quantity taken from the column of rise or fall, as the case may be; and this process of addition or subtraction being repeated throughout the entire column, the reduced height corresponding to the last station will be higher or lower, as the case may be, than the height corresponding to the first station, precisely by the difference between the sums of the back and fore sights, or of the rises and falls. In our example 21·34 is assumed as the arbitrary elevation of the starting point (a bench-mark) above the datum line, and

6·70, the difference referred to, indicating a rise, is added to the first number, and gives

$$21{\cdot}34 + 6{\cdot}70 = 28{\cdot}04;$$

28·04 is therefore the height of the last point above the assumed datum line, and all the intervening numbers indicate the relative elevation of the corresponding point above the same datum.

It is important not to neglect in any instance the means we have indicated of checking the accuracy of the additions or subtractions; for these being in a great measure mechanical operations, there is a liability to error when many pages of the field-book, or long sections, are to be reduced rapidly.

The elevation of the starting point is assumed, as we have said, at an arbitrary quantity. When this starting point is not at a lower level than all the succeeding points, it is desirable, in most cases, to choose a number high enough to avoid tho necessity of using negative quantities for the reduced heights that follow, as they would to some extent tend to confuse the work, and an accidental omission of the sign minus would lead to serious errors in the reduction and plotting.

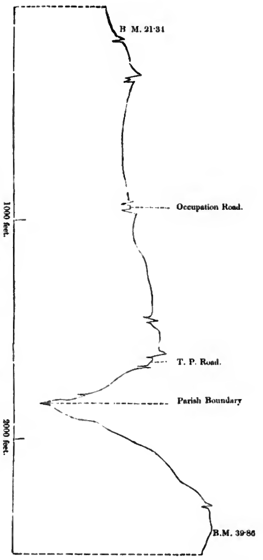
After a section line has been levelled, a second series of levels should *always* be taken from bench-mark to bench-mark, to check the previous work. In this case it is, of course, unnecessary to chain the distances; all that is required is to ascertain the relative elevations of the bench-marks; if these be found the same in the first and second operations, the work is correct: if any sensible difference exist, that part of the section line included between the bench-marks that do not correspond in altitude must be levelled over a third time, in order that the error may be detected and corrected.

The plotting of the section is performed in a manner similar to the plotting of a plan: the elevation and position

of each point that has been levelled are determined by means of two ordinates at right angles to one another, the horizontal distance being measured along one ordinate, the vertical distance or height along the other. Each point having been marked with a fine-pointed pencil, they are all joined by straight lines as the plotting proceeds. The accompanying section is plotted from the data given in the form of the field-levelling book, page 158.

Unless it be in a few exceptional cases, in which sections are plotted on an exceedingly large scale, it is the custom to exaggerate the vertical scale or height, in order to render more prominent to the eye those particular dimensions, forms, or irregularities of the ground, which the section is especially intended to exhibit. Sections over the general surface of cultivated country in lowland districts, if plotted to a true scale, vertically and horizontally, of 4 inches to the mile (the horizontal scale required for sections of roads, railways, or canals to be deposited with the Houses of Parliament), would frequently appear almost as straight lines, and certainly fail to indicate in a striking manner heavy cuttings or embankments that might be requisite in such lowland countries to construct projected lines of communication. The accompanying section is plotted to a horizontal scale of 400 feet to the inch, and to a vertical scale of 20 feet to the inch; the vertical height is therefore exaggerated 20 times.

For the purpose of receiving the plotting of sections, a paper is prepared, on which are engraved faint lines, dividing its dimensions horizontally and vertically into twentieths of an inch, *i. e.*, with lines ruled parallel and at right angles to each other, at the distance of $\frac{1}{20}$th of an inch. Much time is saved by the use of this section paper, as no scale is required for the plotting; it has also the advantage of facilitating correct and rapid measurements of any particular part, the measurement being made by simply counting the engraved lines or divisions, instead of applying



Datum Line, Summit Level of Grand Junction Canal.

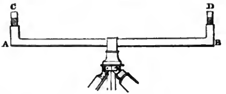
Horizontal Scale, 400 *feet to* 1 *inch*. Vertical Scale, 20 *feet to* 1 *inch*.

a scale. Any regular contraction or expansion of the paper, moreover, does not affect the accuracy of such measurements, the scale being itself embodied on the paper.

Before we conclude the subject of levelling with the spirit-level, we would observe that the late improvements made in the construction of the instrument, and in the mode of reading the levelling staves (improvements due, in a great measure, to the special attention paid to levelling for railway sections), may be said to leave nothing to be desired in point of accuracy and expedition. And we would repeat, that for the purpose of taking sections, the theodolite or other angular instrument used for levelling by means of angles of elevation or depression, is inferior to the spirit-level. In the third volume of the *Trigonometrical Survey*, Captain Mudge, in giving an account of the operations of the year 1797, states that the height above the sea of a station on Trevose Head, a promontory on the northern coast of Cornwall, was levelled with the transit instrument;—the same which had been used in taking the angles of inclination on the Salisbury base. "The height of the station above low-water mark was found to be 274·2 feet; which is, probably, within 6 inches of the truth." The ascent from the sea to the station is further described as being very gradual and unobstructed; under such circumstances, the possession of more suitable instruments would, in modern levelling, enable the difference of elevation to be obtained certainly to within ¼ of an inch of the truth.

The *Water-Level**. Circumstances may occur in which the surveyor or engineer would find it necessary to take a section without having a good instrument at hand. He might, in such a case, use the water-level, an instrument that may be readily constructed in a very short time. It consists simply of a cylindrical tube A B, (usually made of tin,) the extremities of which are bent at right angles to

* MALORTIE's *Topography*, vol. ii., p. 59.



the length of the tube, and support two cylinders C, D, of very transparent glass, open at both ends. When the instrument is to be used, water, slightly coloured in order to define its surface more clearly, is poured into one of the cylinders, and immediately communicates with the other by means of the tube A B. When the water is in a state of rest, the surfaces in each glass tube are on the same level, and the instrument requires, therefore, no adjustment. In making an observation, the surveyor places his eye in the line of the two surfaces of the water, and the intersection, with the staff, of this visual line, gives the reading required. A staff with a sliding vane must be used for this operation, because the observer being unassisted by a telescope, would be unable to read the divisions on the staff, except when placed very near to the instrument. This level, when used in calm and clear weather, is capable of giving results with tolerable accuracy.

CHAPTER VII.

ON SURVEYING AS APPLICABLE TO THE COLONIES *.

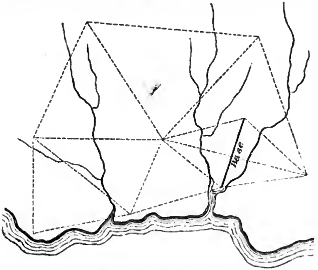
Colonial surveying is distinguished from the usual land surveying, previously described, by a marked difference in its objects. In cultivated countries in which every portion of the land is claimed by a proprietor and an occupier, and the surface of which is divided into estates with known boundaries, or separated into legal and ecclesiastical divisions, the business of the surveyor consists in making on a plan, a faithful representation of the existing demarcations and artificial objects, as well as of the natural features, and in collecting and arranging all data which may contribute to convey a knowledge of the physical aspect of the country. In new colonies, on the contrary, the first purpose of the surveyor, instead of being directed to the measurement of existing lines or boundaries, consists in actually setting out on the ground the limits of stated quantities of land or "sections," previously to their being conveyed to the purchasers.

Bearing in mind this difference, we proceed to describe a suitable mode of conducting such a survey.

When treating of trigonometrical surveying, it was explained that by it only could perfect accuracy be attained in the survey of an extensive district; but, at the same time, the description of the mode of operation made it manifest that it necessarily involves, in its prosecution, both considerable expenditure of money, and great consumption of time. In a new country, probably covered with timber

* We are indebted for the information embodied in the following chapter to the able report *On Surveying as applicable to the Colonies*, made by Captain Dawson, R.E., to the Secretary of State for the Colonies, 1840.

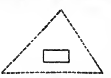
or dense and tangled underwood, intersected by impassable rivers, and inaccessible marshes, and presenting other serious physical obstacles, the consumption of time and money must be proportionably increased. Yet, assuming the trigonometrical survey to be accomplished, and the interior detail of the natural features, such as rivers, streams, mountain ridges, as also the boundaries of coasts, &c., to have been measured and protracted, nothing will yet have been done to assist in the immediate purpose of location; not a single allotment or section of land will have been set out preparatory to its being conveyed to, and occupied by, the purchaser. And thus, the preliminary operations cannot as yet be made subservient to the immediate wants of the settler, the lines defining the boundaries of each location still remaining to be set out on the ground.



This is illustrated by the above diagram on which are shown the results obtained by the Trigonometrical Survey. "From it, it appears that, although much has been done for the connection of isolated surveys, and for

laying them down truly in position as component parts of a general map: the points and lines determined by the Trigonometrical Survey, at this stage of its progress, are not those which are required for the immediate purpose of location.

"If, for instance, it were required to convey a section on the banks of one of the rivers shown in the above sketch, it would still be requisite to trace and measure the lines of the section upon the ground—and if the chosen section were in the middle of one of the triangles, where no lines



have yet been measured, the trigonometrical operation would be altogether unavailable for determining its position, until its limits were actually set out on the ground."

The resources of a new colony are evidently unable, on the one hand, to bear the burden of the expenditure necessary to obtain perfect accuracy; and, on the other hand, its thriving condition must be injured by the delay necessarily consequent on the operation.

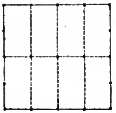
Perfect accuracy must, therefore, in the first instance be sacrificed to economy; and a method adopted capable of providing for the immediate division of the surface into suitable allotments, with an approximate accuracy sufficient for the exigencies of the moment, and attained at the least possible cost.

The square, or rectangle, admitted to be the figure best adapted for the subdivision of lands, is found to lend itself the most readily to such objects. The size of these rectangular divisions or sections must depend on the means of the settler, and the agricultural capabilities of the soil. In Lower Canada the minimum size of the sections has been fixed at 200 acres; that adopted in South Australia contains 80 acres; and the greater number of the purchases in South Australia having been made of sections of the smallest size admitted by the regulations, it may be inferred that the

average means of settlers would be more readily met by the smaller sections. Whatever may be the dimensions adopted, the mode of operation will be similar: it will be the first object of the survey to provide for setting out on the ground the limits of such sections, in the district selected as the most suitable for immediate settlement.

The lines forming the boundaries of the rectangular sections are generally ranged in the direction of the cardinal lines: this ranging should be performed with the theodolite, the surveyor bestowing special care on the reading of the right angles, and never omitting to test the correctness of his work, by measuring the angles formed at the intersections of the boundary lines. The meridian line may be obtained as described in page 118 by observations of the sun: or, as the surveyor has, in such operations, frequently to pass his nights in the field, he may check the direction of his meridian lines by observations of the pole-star.

The directions of these lines or boundaries of sections are ranged by the surveyor himself, while labourers are employed in clearing the lines, by cutting the brushwood or underwood to a width of about 3 or 4 feet. When large trees impede the progress of the line, they are passed as described in page 19. As the clearing proceeds, the boundaries of the sections are marked by strong pickets driven into the ground, at short distances of a few hundred feet, and projecting above the surface not less than from 2 to 3 feet: the bark should also be taken off in order to render them more easily recognized. At the angular points of the sections, *i. e.*, at the intersections of the boundary lines, three or more pickets should be driven in order to distinguish especially those points in the boundaries. If the minimum size of the sections adopted be 80 acres, these pickets would be driven at every quarter of a mile, or at every twentieth chain of 66 feet. If the cardinal lines we have described have been ranged at a distance of 1 mile from each other, they will, by their intersections,



divide the surface into rectangles containing 1 square mile, or 640 acres. These larger rectangles would afterwards be subdivided into the assumed sections of 80 acres each, by ranging lines corresponding to the dotted lines in the figure, and driving interior section pickets at their points of intersection.

"It would be difficult, if not impracticable, to make any arrangement, as the running survey proceeds, for adapting lines of road to the physical features of the country. Such an arrangement will suggest itself, and be found more practicable as the country becomes better known; and in the mean time the lines which bound the eight-section blocks will be found most desirable for the purpose, a convenient and proper space being allowed, and set off, on either side the line." By reference to the above sketch, it will be seen that means of access would thus be afforded to each of the eight sections.

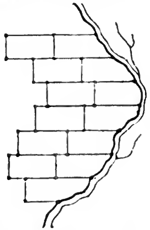
Varying circumstances of locality and physical aspect will necessarily occur to modify the uniformity of this system of survey, demanding occasional departure, both from the direction of the cardinal lines, and from the average size of the allotments, in order that they may be adapted to the natural features of the country. In this adaptation, the element of greatest influence, is the direction of the coasts or of rivers and streams; for, looking to the great advantage which water frontage affords, the object must be to distribute that advantage as equally and generally as possible throughout the settlement. "It becomes, therefore, a question of importance to determine the proportion which should be observed, in water sections, as to frontage and depth. The proportion in the land sections, as above described, is that of 2 to 1, the sides being half a mile, and a quarter of a mile, respectively.

"In the water-sections the proportion may be increased, say as 3 to 1, or as 4 to 1."

In the former case, with the quarter mile frontage, the sections would then contain 120 acres, and in the latter 160 acres, each;—and if the water sections were invariably made of the same size, persons of moderate means might in either case be precluded from their purchase.

In the last of the cases above mentioned, if the double section were to be divided into two of the ordinary size of 80 acres, by a longitudinal line reducing the frontage, the sections would then be a whole mile in length, with a frontage of only 10 chains or 220 yards.

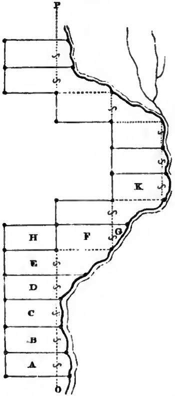
A farm or section of such a form could not be deemed an eligible one: and much less so, if regarded with reference to a "probable future extension of the holdings" in the back direction.



"Preserving a uniform size in the water-sections, would further be attended with the inconvenience of causing the sections to run in steps, as shown in the annexed sketch; and the advantage of a continuous line at the back of the sections would be lost.

"A preferable course under all circumstances, would

be to vary the size of the water-sections, in the following manner, and subject to the following regulations:— The first line should be run from a point O, not more than 10 chains from the river, and the broken spaces between the line and the river should be added to the sections immediately behind them.



"Thus, the first section, A, would be increased from 80 to about 100 acres; the second, B, would be something less, say about 90; the third, C, only a trifle more than 80 acres; the fourth, D, about 100; E, 150.

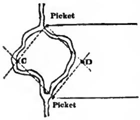
"At F the deflection of the river is such as to admit

of an entire section, of 80 acres, being set out between the main line and the river, together with a broken portion G. F and G, taken together, will therefore form the water-section in this case, and H will be a land-section of 80 acres only.

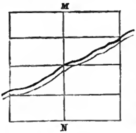
"The same system would be pursued if the river fell off to the extent of two ordinary sections from the line, as at K, the water-section consisting always of the broken portion in addition to the next entire section. In both these last-mentioned cases, however, the measurement would be discontinued along the line O P, and carried along the parallel line which separates the broken portion from the entire section, as shown by the dotted line in the sketch.

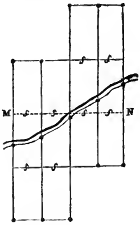
"Thus the water-sections, while in some cases only a trifle larger than an ordinary section, would in others be nearly, though never quite, equal to twice that size, and would never exceed one mile in length. They would be suited, therefore, to the means both of large and small capitalists.

"To determine the actual contents of the water-sections, a survey of the line of the river, or coast, would be required; but if there were no time for this operation before the conveyance of the land were completed, a tolerable approximation to the quantity might be made thus:—The distance between the two frontage pickets on the river bank being only a quarter of a mile, or 440 yards, the direction of the river at some intermediate point, about half-way between the pickets, may be observed with the theodolite, sufficiently near at least to show whether the river runs by C or D, and so to guard against any great deception in the estimated contents of the section.



"The arrangement for varying the size of the water-sections has, moreover, the important advantage of removing entirely the difficulty which might be expected to arise, from laying out the sections arbitrarily, by cardinal lines and parallels, without reference to the natural lines of the country. If, for instance, in the survey of the square mile, represented in the annexed sketch, a river should be found to traverse it, passing through two sections only, giving to those sections an undue allowance of water frontage, and conferring advantages on them, to the exclusion of the other sections: the injustice and irregularity would be found to admit of an easy remedy as the survey proceeds, (without adding materially to its expense,) by merely changing the direction of the sections, measuring the line M N in an east and west direction, instead of north and south, and adding the broken portions of sections to the sections immediately behind them."





If the directions of the lines be fixed by means of the divided limb of the theodolite, and preserved by careful ranging, approximate accuracy may be attained to within certain limits. And if an allowance be made to the purchaser, ample enough to cover casual errors, the possession of the full quantity of land he has paid for will be thereby secured to him; and the trigonometrical operation might consequently be deferred until ulterior objects should call for greater accuracy, and the colony be better able to pay for it.

The allowance would involve a present sacrifice of land, a loss of little moment as compared with the immediate saving of the great expenditure involved in the execution of a trigonometrical survey. As to the amount of this allowance for errors, "3 per cent. would be ample;" and this, added to a further liberal allowance of 2 per cent. on account of public roads, would reduce the purchase-money 5 per cent. on the total amount of land specified in the conveyance.

The map representing the sections, would not be held as giving a faithful copy of their exact form as set out on the ground, but would represent them as true rectangles, whatever might be the deviation from that form in the sections actually set out on the ground. On the sides of the rectangles, the place of the intersections of such rivers, streams, and other natural features as have been crossed in the regular progress of the survey would be marked, but their course in the intermediate areas would be for the time left unmeasured; and thus a skeleton map only would be formed. The allowance made to the purchasers, by securing them from loss, would render the probable distortion of the sections unimportant at first; and when future exigencies should demand accuracy, their true form and position could, by means of an ulterior trigonometrical survey, be determined, and protracted on a map, with sufficient accuracy to constitute it a just record, and conclusive evidence of the boundaries of property.

In the prosecution of this work, the surveyor would require the assistance of 2 chainmen, 2 woodmen, and 2 or more labourers according to the local difficulties of the district: with proper assistance, he could, in dense underwood or brushwood, set out a distance of about a mile per day. The Gunter's chain of 66 feet would be the most convenient for such operations, because, 80 such chains constituting a linear mile, it lends itself readily to the binary division.

The necessity for economy and rapidity in the performance of surveys in all new colonies has been so strongly felt, as to lead sometimes to too wide a departure from accuracy, in order to obtain the objects in view. Time has been economized, first, by making the sections much larger than those we have described; secondly, by using less precise instruments, and bestowing less care in the ranging and measurement of the lines. By adopting the larger allotments, fewer linear miles have to be measured per square mile set out, and time is economized; but, independently of the consequent exclusion of settlers whose means are unequal to the purchase of large sections, this course diminishes the number of checks on the work, and, by causing the accumulation of errors to be distributed over a more extensive space, renders their detection and correction more laborious and intricate, and thereby tends to induce their being passed over unregarded and unremedied.

Under the second head, the use of the compass as the sole means of obtaining the direction of the lines, necessarily introduces serious errors, which are again increased when the lines are not properly ranged by staves fixed in a straight line with a transit instrument or a theodolite. Such has been the practice in North America, where the proposed direction is determined by means of the compass. Also, the lines are there set out without a telescope, simply by the aid of sights raised on the instrument, and instead of

being ranged by staves or rods, they are ranged by the bearing of large trees which stand in the line of sight.

It is obvious that great want of precision must result in ranging a line, when the only object to determine its direction is a tree of considerable diameter. Yet the errors due to this cause are not so considerable as those consequent on variations of the compass, which have been known to exceed half a degree in the course of a day.

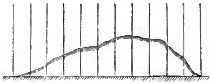
CHAPTER VIII.

HILL DRAWING.

All objects marked in position on an outline map, are protracted according to the laws of the orthographic projection; in other words, their respective distances are reduced to a common horizontal plane. But as surveying aims at making a faithful image of the surface of the country, the map remains incomplete if the rise and fall of the hills, the undulations of the surface, and its whole relief be left unrepresented. In the same manner that a certain disposition of lights and shadows can convey to the mind an idea of the familiar objects around us, so can a physical representation of a hilly surface, by means of light and shade, pourtray the form and height of the hills, the depth and direction of the valloys, and the massive external characteristics of the natural stratification of the country. The only difficulty arises from this circumstance, that the representation of the natural features must be made as if seen from above, a point of sight to which the eye is not accustomed; for in the limited view which it can embrace of objects around, the effects of perspective always present the *apparent* different from the *real* forms. Plans should, if possible, be so shaded and finished that an inspection of them should make the observer acquainted with the relief of the ground, as it would appear in a reduced model of it placed under his eye.

The side or slope of a hill, being inclined to the horizon, receives a smaller proportionate quantity of vertical light than the summit, or than a horizontal plane at its base. A horizontal surface receives an equal portion of light with the inclined surface resting upon it, and as the inclined

surface is of greater extent, it will be darker than the horizontal, in proportion to the degree of inclination and



consequent increase in the extent of surface. A sheet of white paper, bent into the form of a ridge, and placed under a vertical light, affords a simple illustration of this effect, the shade becoming darker, as the inclination is increased*. From this is derived directly the principle that in the representation of the varied forms of ground, the shade applied to the sides of hills should be proportionate to the steepness of the slope. The draughtsman thus gives a physical representation of the hills, generally intelligible to all, and enabling the engineer or professional man to estimate the relative steepness of the hills from a comparison of the relative intensity of the shades.

Height, as well as steepness, is a characteristic to be attended to in the representation of hills. On looking at a reduced model of ground, the highest point attracts the eye in the first instance; in the same manner it should retain its prominence in its representation on the plan, and all differences of altitude should have their relative importance in the drawing. This is effected by combining height with steepness as the two elements to regulate the depth of shade. This combination produces a correct physical representation, by imitating the effects of aërial perspective. The higher points of the country, being nearest the eye, are supposed to present a greater intensity of light and shadow, both of which diminish in degree as

* MITCHELL'S *Outlines of a System of Surveying*, page 69.

the surface becomes of less altitude, because of the supposed intervention, between the surface and the eye, of a greater body of the atmosphere. Through this cause the valleys are slightly shaded or tinted, and can never be confounded with the tops of the highest mountains, which alone are to be left quite white. Were this principle unattended to, the flat parts of valleys, and the level summits of mountains, being both left unshaded, could not be distinguished from one another, and the true appearance of relief would be lost.

In the representation of ground, all features, whether of primary or subordinate importance, whether high mountains or small ridges, should be represented as fully as the scale will admit; but each feature, as in nature itself, or in a model, should be in proper keeping, the smaller features being kept subordinate to those of greater magnitude. The importance of preserving each feature or object in its proper "keeping" is well understood in drawing or painting when cultivated as a branch of the fine arts, and it is one of the elements most conducive to beauty and truth of representation. When unattended to, the drawing presents a harsh effect of spottiness. Until of late years it was altogether neglected in topographical drawing; high mountains, howsoever extensive their base might be, or however varied their ramifications into other subordinate forms, were represented much in the same manner, and with the same tone, as the inferior hills of the low country.

Mr. Dawson, for more than half a century connected with the Topographical Department of the Ordnance Survey of England and Wales, was the first to correct this defect, by establishing the following general principles.

1. That a plan must be considered as a full face portrait of a country.

2. That mountains, hills, and hollows must be considered as features varying the general face of the ground.

3. That every feature must be conceived and expressed

as a whole object; that is, according to its effect on the eye as a whole form.

4. That features must be drawn according to their assembled effect as a general whole.

And as to Mr. Dawson was entrusted, for many years, the instruction in hill drawing of the officers of engineers on their leaving Woolwich Academy, the conventional representation of hills, as so many isolated features, all partaking of the same character, was by degrees abandoned, and the improved system, which aims at copying nature both in the great masses and the minute details, was generally introduced.

Different methods of shading, and various dispositions of lines are used to give the physical effect of relief to a map: of these methods, that which supposes the light to fall vertically on the ground is well suited to give the physical representation required, and, by causing the intensity of the shade to depend on the height of the hills and the relative steepness of their sides, it is best adapted to offer a precise and uniform method of ascertaining these important characteristics.

The physical effect of shading can be produced by etched lines; when this method is adopted, the lines may be drawn on the plan in the same relative position that would be occupied by lines traced on the ground parallel to the horizon. Such lines are called horizontal contours. If we imagine horizontal lines vertically equidistant, that is, each line separated from the adjoining lines above and below by a given constant altitude, they will, when projected orthographically on the plan, necessarily approach, or recede from, each other, according as the slopes are more or less steep; on ground of slight inclination they will be distant from each other and produce light shades, whereas on steep slopes they will approach closer to each other, and consequently produce a depth of shade proportionate to the steepness of the slope. If a series of such horizontal

lines, all vertically equidistant, be traced from the base to the summit of a hill, they will present a correct geometrical and physical representation of the hill; and, the vertical distance which separates the contours being known, a profile or section can be drawn in any direction whatever.

The idea of employing horizontal lines for the representation of forms suggested itself, as early as 1738, to Philipo Buacha, when watching, by a retreating tide, the successive intersections of the plane of the surface of the sea with a shelving and varied coast. But he only aimed at tracing them as lines of equal soundings on hydrographical charts. Thirty-three years later, Ducarla proposed to adapt imaginary lines, following the same law, to the representation of the features of ground; and, in 1782, Dupain-Triel gave stability to the principle, by introducing the important element, that the lines should all be vertically equidistant*. However, owing to the want of precise data, and accurate outline maps, the principle was not fully carried out until of late years, when these vertically-equidistant horizontal lines, which we shall call *normal contours*, came to be determined in the field by means of the theodolite, spirit-level, or other instrument. These normal contours have been adopted in the French cadastre since 1818†, and are now also adopted in the Irish Survey, in which latter work their introduction dates from the year 1838. These contours, for the sake of clearness, and to avoid unnecessary labour, are kept at a convenient vertical distance from each other. No general rule can be given for the distance to be adopted; it must vary according to the scale of the plan, and the nature of the country to be represented. If the scale be small, the vertical distance on the ground must be increased, and

* PUISSANT'S *Figuré du Terrain*, p. 81.

† *Ibid*, p. 67; also, *Instructions données en* 1818 *aux Ingénieurs Géographes par l'Administration du Dépôt de la Guerre.*

if the country be very mountainous and rugged, with steeper slopes to the hills than are met with in the lower cultivated grounds, the time and labour required for tracing horizontal lines, separated by small vertical distances, would be incommensurate with their practical application; such a country being usually uncultivated and barren. In such localities, therefore, the contours may be traced at wider intervals.

The vertical distance adopted on the Irish Survey for the scale of 6 inches to a mile, or $\frac{1}{10560}$th of the actual size, is 50 feet for the cultivated part of the country, and 100 feet for the mountainous districts. On the French Survey, the plans of which are drawn to a scale of 400 centimètres to a metre, or $\frac{1}{20000}$th of the actual size, the vertical distance is 10 metres. On the special plan made of Paris and its environs to a scale of $\frac{1}{5000}$th of the actual size, the vertical distance between the contours is only 2 metres.

The normal contours, traced by means of an instrument, being too much separated to produce a shade capable of giving a forcible representation of the hills in relief, other lines parallel to the normal ones are introduced in order to constitute a shade. This part of the work may be made mathematically correct, by assuming that a given number of lines of the same thickness shall always be introduced between the normal contours, which lines, by being brought close together on steep slopes, will there produce dark shades, and lighter shades on gentler slopes, being separated in the ratio of the variation of inclination. It would be extremely difficult, and it is unnecessary to follow that law with geometrical precision. The normal contours will guide the eye and hand in giving the proper depth of shade by means of intermediate etched lines, freely drawn parallel or nearly so to the normal contours; which contours will be sufficiently close to give a correct section of the ground by combining the measured and *varying* horizontal distances

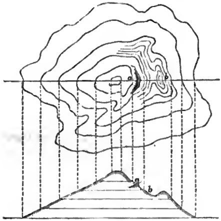
between the contours, with the *constant* vertical height which separates them.

On the French cadastre, those who trace the normal contours in the field, have instructions not to introduce any intervening horizontal lines on the plan. The physical relief is afterwards produced in the office, by using the normal contours as guides for the intensity of shade.

On the Irish Survey the same preliminary process is followed, but with this important ulterior improvement, that the maps, with the normal contours traced upon them, are afterwards placed in the hands of field-parties, whose duty consists in filling in the detail to produce physical relief, and give expression of form and character to the drawings.

To obtain accuracy, this last process is indispensable: for the vertically-equidistant contours frequently omit important intervening features, as is shown in the adjoining plan and section, wherein the rocks at a, and the back fall

Plan.



Section.

at b are left unrecorded, simply because they present themselves between the normal contours.

The normal horizontal lines may be traced on the ground by means of the theodolite, the spirit-level, water-level, reflecting-level, or any traversing instrument; but before these contours can be traced, an accurate outline map of the ground must be constructed.

The contours may be traced thus. Ranging rods should be fixed at certain distances along the ridge or water-shed lines, and along the valleys or water-courses, and such other natural lines as best define the undulations of the surface; and the positions of these rods must be marked on the plan. A short picket being driven between two of the ranging rods at the proposed level of one of the contours to be traced, the theodolite, or, in preference, the spirit-level, is placed at a convenient distance from the picket and at such an elevation that, when adjusted horizontally, the line of sight shall intersect a levelling-staff placed on the picket. For this purpose a staff with a sliding vane must be used, as it admits of being observed at greater distances than the improved levelling-staves. The centre of the vane is, by attention to signals, brought by the staff-holder to the point of intersection of the visual ray, and fixed in that position; the staff is then taken to a point nearly on the same level between the two next ranging rods, and moved up or down the slope until the centre of the vane again coincides with the horizontal line of sight. A picket is driven at this spot, and the operation is continued round the valley or brow of the hill as far as the observer can see the staff distinctly: the position of the spirit-level is then changed, and the same horizontal line continued as far as required. The next contour, either above or below, is traced in exactly the same manner at the required vertical distance from the first. While sweeping the telescope of the spirit-level from point to point, the observer takes care to note where the visual ray intersects any object, such as the base of a house, gate, corner of a field, &c., which is marked on the plan, and he thereby

obtains the elevation of an additional number of points. The positions of the others that have been observed, as marked on the ground by pickets, are fixed by measuring with the chain their distances from the ranging rods whose positions are known. Finally, the normal contours are traced by joining (by lines nearly straight, but partaking of the general roundness of the ground) the various points indicating the same elevation. It is almost needless to observe that fixed points must be carefully levelled at different distances to serve as bench marks for the detection of errors that would unavoidably accumulate, were contour lines to be carried great distances without checks from independent data.

This method of tracing normal contours may also be followed, by using, instead of the spirit-level, the water-level, or French reflecting-level. The principle is the same with all; but these last instruments cannot be recommended for the purpose, as they are not so accurate, and save but little time.

A map with such contours accurately traced upon it, is of much greater value to the engineer than if left simply in outline, inasmuch as he can trace sections in any direction, by measuring the horizontal distances along the line of section between each pair of contours, the vertical distances being a known constant quantity. But the process of tracing such normal contours in the field, is so laborious, that it can be carried into execution on a large scale only as forming part of a great national survey, which then becomes a fit guide and basis to facilitate and direct the projection of internal improvements.

Horizontal contours can be traced by the eye with considerable accuracy, especially when the surveyor is assisted by the altitudes obtained in the trigonometrical operations serving to the construction of the outline map. The process before described is, from its mechanical nature, slow; that which we now proceed to describe is rapid in execution,

and tolerably correct for a small scale (say 1 inch or 2 inches to a mile), where experience has trained the eye to accuracy. In the field, when the eye alone is depended upon, the horizontal lines are traced in pencil, by close parallel hatchings; and when the whole drawing is finished the normal contours are traced at the required vertical distances apart, by following the general direction of the pencil lines, and checking their truth by means of the trigonometrical elevations marked on the map. The contours, when a complete circuit is made, must return to the point of departure, and if it were attempted by the eye alone to trace normal contours which are isolated from each other, scarcely any degree of previous experience could suffice for the attainment of the object.

The practice has long prevailed of transferring to the fair drawing or to the copper plate for engraving, not the horizontal lines as sketched in the field, but vertical lines at right angles to them, which represent the course that would be followed by water in its descent down the slope. This style of hill-drawing is called the "vertical style," in contradistinction to the first described, which is called the "horizontal style." The vertical style is rarely used for the field work; but whatever style may be adopted in the field, the maps when engraved have hitherto been etched in the vertical style.

It seems difficult on first consideration to account for this substitution of vertical lines for the horizontal lines, as originally drawn in the field, especially when it is considered that any change must diminish in the copy the value due to the original document. Also, for engineering purposes, the change is inconvenient, as the horizontal lines must again be restored by tracing them at right angles to the vertical ones; and such a change cannot be made without introducing errors. The chief cause of the alteration seems to be that the vertical lines can be etched by the draughtsman, and more especially by the engraver, with

greater facility and rapidity than the horizontal lines: it is believed, however, that, to a certain extent, the greater facility of engraving in the vertical style is due to the artists having exclusively practised in that style.

For a small scale, such as that of 1 or 2 inches to a mile, when it is intended to introduce into the plan all the details that the scale admits of, etching with the crow quill for the fair plan is to be preferred. But for plans aiming at less exactness of detail, or prepared on a larger scale (from 3 inches to a mile and upwards), washing in the tints with Indian ink is much preferable, as it admits of greater rapidity of execution. The field work is, nevertheless, finished as before described in pencil, and normal contours are, if possible, traced and drawn with the pen; the tints of Indian ink are then laid on according to the rule of the depth of tint being proportional to the height of the hills and the steepness of the slopes. To soften the tints, when it is required to represent a rounded form, two hair pencils are used, one as the colour brush, the other as the water brush. The shades are laid on with the colour brush, and softened by passing the water brush rapidly along their edges. The water brush should not have much water, as it would in that case lighten the shadow to a greater extent than is intended, and leave a ragged harsh edge. Tints may be rounded without softening the edges with the water brush, by using very light colour, and applying one tint over another, with the boundary of the upper tint not reaching to the extreme limit of the tint below it;—a beautiful effect of clearness and transparency is by this means given to the drawing. Also, when depth of shade is required, it is best produced by the application of several light tints in succession; for when the full depth is given by a single wash, its effect is rough and opaque. No tint is to be laid over another until the first is dry; and a little indigo mixed with the Indian ink improves its colour, and adds to the richness of effect.

Hullmandel's valuable discovery in lithography, which enables the artist to use the brush on the stone with as much freedom and effect as on paper, will no doubt be made available for the representation of hills in plan, and will lead to the publication of highly-finished drawings on a larger scale than is suitable to the more laborious process of etching on copper.

The accompanying plate, the first topographical representation of hills executed in lithotint, is copied from an original drawing by M. Caplin, whose merits as a skilful topographical artist are well known and appreciated.

There are numerous advocates for the representation of hills, under a supposed "side light," the light in that case descending obliquely at a fixed angle from one side of the map. The sides of the hills next the light receive it more or less brilliantly, according as they are inclined more or less nearly at right angles with its rays; and the shade on the sides removed from the light increases in intensity as the slopes increase in steepness. This style may be rendered most expressive by a skilful draughtsman, especially when the character and strike of the hills is favourable to the direction of the light; this is the case when the strike of the hills is at right angles or nearly so to the direction of the light, and when the steepest sides of the hills are uniformly on the shaded side. Such a disposition of the forms of ground can only obtain over a very limited space, and when the map comprehends an extensive district, the steepest sides of the hills will in some cases be opposed to the light, in others range in directions parallel to it, and in both circumstances the less inclined slopes will receive the darkest shading, and may mislead as to the character of the country. With this style of representation, the hills are generally made to partake more or less of the same character, appearing almost uniformly steepest on the sides removed from the light. It is a disadvantage inherent to this method, that by it much more scope is left to the

taste of the draughtsman, and the topographical language thereby loses some part of its universality; also, if, on looking at a map so drawn, the light whereby it is examined should happen to fall in a direction contrary to that in which it is assumed to proceed, the effect of relief is, to some extent, diminished; and the writing and outline necessarily introduced on the plan contribute also to mar its effect. Most beautiful drawings of the mountainous districts of Wales have been executed for the Ordnance Survey in this style; and they could not probably be surpassed in truth of execution, or in pictorial effect and breadth of expression. There can be little doubt, however, that artists equally skilful could give as rich an effect by means of the vertical light, and if so, the latter style should seem to claim the preference; for maps so drawn may be considered as furnishing more precise data, the relative height and steepness of the hills being represented by corresponding depths of shade, and the writing and outline interfering less with the general effect.

The roads must be left either untinted or lighter than the adjacent shade, in order that they may be easily seen on the map. When woods have to be represented, the shading used for the trees, instead of interfering with the shadows due to the slopes, may be made to harmonize with them, and to contribute to the general effect by presenting greater or less depth, according to the position of the woods on the sides or summit of the hills.

The extended adoption, in some parts of the continent, of the style of hill drawing, known as the *German System*, renders it necessary that some reference should be made to it. The fundamental principle of the system (which may be applied equally to the horizontal or vertical style) is, that the depth of shade shall be mathematically proportionate to the angle of inclination. This system differs from that which we have described in two particulars;—first, the omission of height as an element in the estimate

of the intensity of shade; secondly, the attempt to make the depth of shade vary with *mathematical* accuracy according to the rate of inclination. By omitting to consider elevation as an element influencing the management of light and shadow, the effects of harmony and keeping are lost, and the physical relief or representation is less perfect: the plan presents an appearance of spottiness, in which the subordinate features often appear as prominent as the great and more important mountain masses. The object of making the depth of shade mathematically proportionate to the rate of inclination may be important, but it is extremely difficult of attainment in practice; and the use of the "anglometer" or "clinometer," for determining the angle of inclination, as proposed by some advocates of the system, would be found inadequate to the purpose, unless the inclinations were first set out with long rods on the ground to which the instrument could be applied. The tracing of vertically-equidistant contours would be the most rapid and accurate way to obtain the inclination required. Supposing it to be practicable on the plan to máke the intensity of the shades strictly proportionate to the rate of inclination, the work thus completed could not be applied in practice with a degree of exactness proportionate to the truth of execution, because the relation of different degrees of intensity of shade cannot be estimated with precision, and does not admit of exact admeasurement; whereas the adoption of normal vertically-equidistant contours solves the problem more correctly, owing to the exactness with which the horizontal distances and the heights can be measured. This system has been advocated and applied in different countries under various modifications, details of which will be found in Sir J. C. Smyth's *Topographical Memoir*, and Lieut. Siborn's *Instructions in Topographical Plan Drawing.*

Topographical Modelling.

The object of a plan is to pourtray nature as correctly as possible: a model approaches more nearly to the truth by affording a substantial representation of the forms. This must not, however, be understood as implying that a model is, in all cases, preferable to a plan. For example, in applying it to projects in which lines of levels are required, a plan giving the levels from direct measurement in the field, by horizontal contours or otherwise, is more serviceable than a model, correctly constructed by scale from the same data, but which data are not written upon the model. For when the heights are to be referred to for purposes of calculation, they must be remeasured from the model with chances of error greater than those presented by the plan. The plan in such a case gives the recorded heights as obtained *from direct measurement in the field;* whereas the model gives them only when deduced by a *secondary* process from measurements on the *diminished* scale; and in proportion as the scale of the model is diminished, so are the the chances of error increased. This objection would not, of course, apply, were the heights, as obtained from the measurements in the field, written upon the model.

Under such a condition, and independently of it also, models are of great value, for the purpose of giving a correct general knowledge of the country, especially to those who have not made hill drawing a study; but their value, as applicable to practical purposes, is greatly enhanced when they are made in parts, capable of being separated, so as to exhibit vertical sections of the geological stratification. Mr. Sopwith, who has directed much attention to this subject, has shown, "by a series of small models constructed of differently coloured plates of wood, the advantage of expressing in a solid form those fractured conditions of the

strata, a right understanding of which is of the greatest importance, both to the working of coal mines and of metallic veins. Many of the complicated phenomena of curvatures and complex intersections of plane surfaces cannot be adequately represented by any kind of geometrical drawings or plans; to the perfect knowledge and economical working of a mineral district, it is essential that the subterranean relations of all the strata should be correctly known and expressed in an intelligible form:—1st. The original order of stratification; 2nd. The amount of dislocation by fracture; 3rd. The changes of the surface produced by denudation: and all these can be intelligibly and simultaneously expressed by models.

"The deceptive appearances frequently caused by faults or fractures are represented by dissecting and making the models moveable in the direction of those faults, so that the strata may be restored to their original position, and again shifted or dislocated. The still further difficulties which arise from the denudation of the upper portion of the dislocated strata, can be adequately expressed only by the solid fac-similes of nature which models afford*."

A topographical model, intended solely to represent the surface of the country, without aiming at conveying a knowledge of its internal stratification, is easily constructed, as follows, from the data given on a correct plan.

Trace the outline on paper, and paste the tracing on a level board. By means of pincers fix wires into the board at the principal stations, and at the points remarkable for their elevation, or serving to give the position of the leading features. With nippers cut off the wires to the proper height, as ascertained by a vertical scale applied at their side. With fine modelling clay fill up the spaces between the wires, and give the true form and elevation to the

* *Proceedings of Geol. Soc.*, Dr. Buckland's Address, 1841, p. 474.

intervening surface, by reference to a hill-sketch in horizontal contours.

When the model is finished, a mould of it in plaster of Paris is prepared in the common way, and when the mould has been completely dried by the heat of the sun or fire, plaster casts may be taken from it. To harden the surface of the plaster in the mould and casts, they are covered with two or three coats of drying oil. If it be required to write, or trace outline, on the casts, they are prepared to receive the ink or colour, by a wash of isinglass or glue sizing, applied when quite hot.

In almost all cases of topographical modelling on small scales, it is advisable to exaggerate the vertical scale, in order to convey to the observer the same appearance as that presented by nature. Under any circumstances of observation, owing to the small elevation of the eye, mountains and hills appear in profile, with the horizontal distances more or less fore-shortened, and consequently apparently diminished, their elevation suffering no apparent diminution from the same cause. When examining a model, on the contrary, the eye is so much above it, that a degree of fore-shortening takes place with respect to the vertical, and none with respect to the horizontal distances; an impression therefore is conveyed different from that which nature presents to us.

Mount Etna clearly illustrates the proposition, that the effect, on our senses, of extension in height is exaggerated in nature. The base of the mountain is about 87 miles in circumference, and its height 10,874 feet, or about 2 miles. Its profile, in a model made in true proportion, would present an elevation equal only to one-fourteenth part of its base; such a model, unless on a very large scale, would not recall to our minds the impressions caused by a sight of its bold features and high relief.

The exaggeration to be given to the vertical distances, depends on the proportion which the model bears to nature.

If the scale be very small (say 1 inch to a mile, or $\frac{1}{63360}$th of the actual size), the height may be doubled; with a scale of 6 inches to a mile, or $\frac{1}{10560}$th of the actual size, the height may be increased by one-half, and so on in proportion, the exaggeration diminishing as the scale increases. The relation must depend also on the nature of the country, a low undulating country requiring more exaggeration than a mountainous and rugged district.

On a small scale this exaggeration is absolutely necessary in order to represent the small features, which would otherwise become microscopic objects.

CHAPTER IX.

LEVELLING WITH THE MOUNTAIN BAROMETER.

Experience having onco clearly demonstrated that, in a barometer carried successively to different elevations above the level of the sea, the height of the mercurial column diminished as the elevation increased, the application of the barometer to tho mensuration of altitude readily suggested itself*. Experiment, combined with observation, further determined the law, that, when the elevation increases in an arithmetical ratio, the weight or density of the atmosphere, and consequently the height of the mercurial column, diminish in a geometrical ratio. If to this be added a knowledge of the real altitude which corresponds to any given height of the mercurial column, (or to any given density of the atmosphere,) the relative altitudes of different stations may be deduced from observations made at each station, by means of the barometer, on the varied pressure of the atmosphere, under the same conditions of temperature.

There are various experiments by which the real altitude which corresponds to any given height of the mercury may be deduced; the readiest is that which results from the known specific gravity of air, with respect to the whole pressure of the atmosphere on the surface of the earth. The investigation is thus given by Hutton†:—

"Because the terms of an arithmetical series are proportional to the logarithms of the terms of a geometrical series, different altitudes above the earth's surface, are as

* Biot's *Traité de Physique*, book ii., chap. 5.

† Hutton's *Mathematics*, vol. ii., page 261.

the logarithms of the densities, or of the weights of air, at those altitudes.

So that, if D denote the density at the altitude A,

and d denote the density at the altitude a;

then A being as the log. of D, and a as the log. of d, the dif. of alt. $A - a$ will be as the log. $D -$ log. d, or log. $\frac{D}{d}$.

And if $A = o$, or D the density at the surface of the earth; then any altitude above the surface a, is as the log. of $\frac{D}{d}$.

"Assume h so that $a = h \times \log. \frac{D}{d}$, when h will be of one constant value for all altitudes; and, to determine that value, let a case be taken in which we know the altitude a corresponding to a known density d; as, for instance, take $a = 1$ foot, or 1 inch, or some small altitude; then, because the density D may be measured by the pressure of the atmosphere, or the uniform column of 27600 feet, when the temperature is 55°; therefore 27600 feet will denote the density D at the lower place, and 27599 the less density d at 1 foot above it; consequently $1 = h \times \log. \frac{27600}{27599}$; which, by the nature of logarithms*, is nearly $= h \times \frac{\cdot 43429448}{27600} = \frac{h}{63551}$ nearly; and hence $h = 63551$ feet; which gives, for any altitude in general, this theorem, viz.:

$$a = 63551 \times \log. \frac{D}{d},$$

or $a = 63551 \times \log. \frac{M}{m}$ feet $= 10592 \times \log. \frac{M}{m}$ fathoms; where M is the column of mercury which is equal to the pressure or weight of the atmosphere at the bottom, and m that at the top of the altitude a; and where M and m may be taken in any measure, either feet or inches, &c."

However, a comparison of the observed heights of the mercury at different stations, is not in itself sufficient to

* See HUTTON's *Tracts*, vol. i., Tract 21, *On the Construction of Logarithms*.

give their difference of altitude; because mercury expanding or contracting with every change in its temperature, the same weight of the atmosphere will not counterbalance, with altered temperatures of the mercury, the same height of the mercurial column. The expansion in mercury due to an increase of 1° of Fahrenheit is $\frac{1}{9600}$th* of its bulk. When comparing, therefore, distinct observations of the barometer, the heights of the column must be reduced to a common standard, by deducting $\frac{1}{9600}$th of the height, for every degree of temperature above the standard. To obtain this necessary element in the calculation, a thermometer is fastened to the tube of the instrument.

A correction is also required for the temperature of the atmosphere, for a similar reason. The density of the atmosphere is greatest near the surface of the earth, and diminishes as the distance from it increases. But by an increase of temperature, air expands $\frac{1}{435}$th* of its bulk for every degree of Fahrenheit. A consequent rarefaction of the column that supports the mercury takes place, and the denser parts of that column are raised higher than they were at the previous lower temperature. Under this circumstance, when the barometer is placed at some elevation above the sea, the mercury has to sustain an additional weight, namely, a weight as great as it would have had to sustain, if brought down in the atmosphere to the place formerly occupied by the denser air now raised above it. It is only at the level of the sea, that the pressure would remain constant. The *same difference* of height of the mercurial column at two stations, indicates a difference of altitude, increasing as the density of the air diminishes by an increase in its temperature.

"The formula above given, $a = 10592 \times \log. \frac{M}{m}$ fathoms, is adapted to the mean temperature of the air at 55°. It may be rendered much more convenient for use, by reducing

* HUTTON'S *Mathematical Dictionary.*

the factor 10592 to 10000, by changing the temperature proportionally from 55°; thus, as the difference 592 is the 18th part of the whole factor 10592; and as 18 is the 24th part of 435; therefore the corresponding change of temperature is 24°, which reduces the 55° to 31°. So that the formula is $a = 10000 \times \log. \frac{M}{m}$ fathoms, when the temperature is 31°; and for every degree above that, the result is to be increased by so many times its 435th part."

As considerable differences of altitude are found to cause only small changes in the height of the mercurial column, it becomes necessary, in order to obtain accurate results, to measure with great precision very minute changes in the height of the mercury. Great mechanical improvements have, with that view, been made from time to time in barometers of different constructions. Among these we select for description, the following construction of mountain barometer, as one of the most exact and convenient hitherto constructed*.

The glass tube, containing the mercurial column, is connected with a glass cistern open to the influence of the atmosphere. A brass casing or tube, to protect the mercurial tube and cistern, is suspended from a tripod stand, by rings or gimbals in the same manner as the mariner's compass, so as to assume by its own weight a truly vertical position. In the upper half of the brass tube, two opposite vertical slits are made, so that the surface of the mercury can be seen against the light. The brass tube is divided into inches and twentieths, to measure the height of the mercury, the zero point being in a plane passing through the upper edges of two rectangular slits made horizontally on opposite sides of the tube, near the bottom. These slits enable the observer to see the inferior surface of the mercury, which should at each observation be made to coincide with

* BIOT'S *Traité de Physique*, and SIMMS on *Mathematical Instruments*.

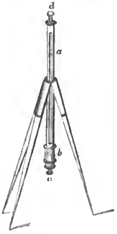
the zero of the scale. The adjustment of the zero point is made by means of the moveable base of the cistern, which is raised or depressed as required by an attached screw,

a *Upper reading.*

b *Zero point.*

c *Screw to adjust the zero point.*

d *Screw to adjust the vernier.*



until the upper edges of the slits exclude the light, by forming a tangent to the slightly convex surface of the mercury. The height of the upper surface of the mercurial column is obtained by means of a sliding vernier, the zero point of which is, by a similar contrivance of the exclusion of light, made to coincide to the greatest exactness with the central part of the convex surface of the mercury. The reading of the vernier then indicates the height of the column of mercury, counterbalancing the pressure of the atmosphere at the time of observation. The vernier is usually made to indicate the minute quantity of $\frac{1}{200}$th of an inch;—the primary scale being divided into inches and twentieths, a space equal to $\frac{9}{20}$ths of an inch is divided on the vernier into 10 parts; consequently a division on the vernier is smaller than a division on the primary scale by $\frac{1}{10}$th, and the vernier reads therefore to $\frac{1}{200}$th of an inch.

For the purpose of indicating the changes in the temperature of the mercury, and the consequent changes in

its bulk, a thermometer is attached to the instrument. As this thermometer forms a constituent part of the barometer, it is not capable of indicating, with sufficient exactness, changes of temperature that affect the atmosphere much more rapidly than they can influence a solid mass like the barometer. To estimate, therefore, the changes of temperature in the atmosphere, an extremely delicate and sensible thermometer is required. When in use, it should be sheltered from the sun, and suspended some feet above the ground in such a position as to allow a free circulation of air around it.

The observed differences of temperature of the attached and detached thermometers constitute, with the difference of height in the mercurial column, the principal elements required to obtain the relative altitude of two or more stations. But in addition to these data, other elements, such as the increase of gravitation from the equator to the poles, hygrometric changes in the atmosphere, periodical oscillations in its density, and other causes, combine to influence the results. Complex investigations, unsuited to the scope of this treatise, have been made by Laplace, Prony, Mr. Bailey and others, with the view to embrace the action of these phenomena in one general formula suited for purposes of computation. Their investigations have led to such accurate results, that under favourable circumstances of the atmosphere, and of locality, barometrical observations can be expected to give results nearly as accurate as those obtained by trigonometrical operations.

Before transcribing the formula obtained as the result of these deep investigations, we shall insert the following simple precepts for the observation and computation of altitudes, given in HUTTON'S *Mathematical Dictionary*. They are deduced directly from the preceding considerations, and from the formula

$$a = 10000 \times \log. \frac{M}{m} \text{ fathoms.}$$

Observe the height of the mercury, and the temperature of the attached and detached thermometers at each station.

Reduce the mercury to a common temperature, by increasing the colder, or diminishing the warmer, by $\frac{1}{9600}$th part of the height, for every degree of difference between the two.

Take the difference of the logarithms of the heights of the barometer thus corrected, removing the decimal point four places more towards the right hand, the figures on the left will be the height in fathoms, to which a correction remains to be applied for the difference of temperature in the atmosphere at the two stations.

This correction is effected as follows:—" Take half the sum of the temperatures, shown by the detached thermometers, for the mean one; and for every degree which this differs from the standard temperature of 31°, take so many times the 435th part of the fathoms above found, and add them if the mean temperature be more than 31°, but subtract them if it be below 31°; so shall the sum or difference be the true altitude in fathoms, or, being multiplied by 6, it will give the true altitude in English feet."

Example.

Barometers.	Thermometers. Attached.	Thermometers. Detached.
29·98	63°	62°
26·17	47	45
	Dif. 16	2) 107
		Mean . $53\frac{1}{2}$
		Standard 31
		Dif. . $22\frac{1}{2}$

9600 : 16 :: 29·98 : 0·05

Correction 0·05

M =	29·93	log.	1·4761067
m =	26·17	log.	1·4178037
		Dif.	0·0583030

Approximate height = 583·030 fathoms.

$$435 \; : \; 22\tfrac{1}{2} \; :: \; 583\cdot030 \; : \; 30\cdot157$$
$$30\cdot157$$

The altitude sought is $613\cdot187$ fath. $= 3679\cdot122$ feet.

The following formula, the result of the complex investigations referred to in page 201, is taken from MR. BAILEY'S *Astronomical Tables and Formulæ, page* 111; and with the assistance of the annexed table, it renders the computations very simple and rapid.

Making $\beta =$ height of the barometer, $\tau =$ temp. (Fahr.) of the mercury, $t =$ temp. (Fahr.) of the air — at the lower station;

$\beta' =$ height of the barometer, $\tau' =$ temp. (Fahr.) of the mercury, $t' =$ temp. (Fahr.) of the air — at the upper station;

$\phi =$ latitude of the place,

$x =$ difference of altitude required in English feet;

then

$$x = \left[60345\cdot51 \times \{1 + \cdot00111111\ (t + t' - 64^\circ)\}\right] \times \log. \left[\frac{\beta}{\beta'} \times \frac{1}{1 + \cdot0001\ (\tau - \tau')}\right] \times [1 + \cdot002695 \text{ cos. } 2\ \phi].$$

The value of x is obtained as follows from the following table; the log. of the two first terms, included in the large vinculum, is obtained by reading from the column A, the number corresponding to the sum of the degrees read on the detached thermometers (t and t'), as given in the column S; the log. of the last term is obtained from the column C, taking the number corresponding to the latitude of the place of observation as given in the column L: with respect to the middle term, the column B of the table gives the log. of the factor $\dfrac{1}{1 + \cdot0001\ (\tau - \tau')}$, by taking the number corresponding to the difference of the degrees read on the attached thermometers as given in the column D, the number in the left-hand column of B being taken when the

TABLE *for determining Altitudes by the Barometer. Computed by* MR. HOWLETT *from the Formula given by* MR. BAILEY.

Attached Thermometers.			Detached Thermometers.								Latitude of the place.	
D.	B.		S.	A.	S.	A.	S.	A.	S.	A.	L.	C.
	Therm. highest at lowest station.	Therm. lowest at lowest station.										
			40°	4·7689067	75°	4·7859208	110°	4·8022936	145°	4·8180714	0°	0·0011689
			41	4·7694021	76	4·7863973	111	4·8027525	146	4·8185140	3	0·0011624
			42	4·7698971	77	4·7868733	112	4·8032109	147	4·8189559	6	0·0011433
0°	0·0000000	0·0000000	43	4·7703911	78	4·7873487	113	4·8036687	148	4·8193975	9	0·0011117
1	0·0000434	9·9999566	44	4·7708851	79	4·7878236	114	4·8041261	149	4·8198387	12	0·0010679
2	0·0000869	9·9999131	45	4·7713785	80	4·7882979	115	4·8045830	150	4·8202794	15	0·0010124
3	0·0001303	9·9998697	46	4·7718711	81	4·7887719	116	4·8050395	151	4·8207196	18	0·0009459
4	0·0001737	9·9998262	47	4·7723633	82	4·7892451	117	4·8054953	152	4·8211594	21	0·0008689
5	0·0002171	9·9997828	48	4·7728548	83	4·7897180	118	4·8059509	153	4·8215988	24	0·0007825
6	0·0002605	9·9997393	49	4·7733457	84	4·7901903	119	4·8064058	154	4·8220377	27	0·0006874
7	0·0003039	9·9996959	50	4·7738363	85	4·7906621	120	4·8068604	155	4·8224761	30	0·0005848
8	0·0003473	9·9996524	51	4·7743261	86	4·7911335	121	4·8073144	156	4·8229141	33	0·0004758
9	0·0003907	9·9996090	52	4·7748153	87	4·7916042	122	4·8077680	157	4·8233517	36	0·0003615
10	0·0004341	9·9995655	53	4·7753042	88	4·7920745	123	4·8082211	158	4·8237888	39	0·0002433
11	0·0004775	9·9995220	54	4·7757925	89	4·7925441	124	4·8086737	159	4·8242256	42	0·0001223
12	0·0005208	9·9994785	55	4·7762802	90	4·7930135	125	4·8091258	160	4·8246618	45	0·0000000
13	0·0005642	9·9994350	56	4·7767674	91	4·7934822	126	4·8095776	161	4·8250976	48	9·9998775
14	0·0006076	9·9993916	57	4·7772540	92	4·7939504	127	4·8100287	162	4·8255331	49	9·9998372
15	0·0006510	9·9993481	58	4·7777400	93	4·7944182	128	4·8104795	163	4·8259680	50	9·9997967
16	0·0006943	9·9993046	59	4·7782256	94	4·7948854	129	4·8109298	164	4·8264024	51	9·9997566
17	0·0007377	9·9992611	60	4·7787105	95	4·7953521	130	4·8113796	165	4·8268365	52	9·9997167
18	0·0007810	9·9992176	61	4·7791949	96	4·7958184	131	4·8118290	166	4·8272701	53	9·9996772
19	0·0008244	9·9991741	62	4·7796788	97	4·7962841	132	4·8122778	167	4·8277034	54	9·9996381
20	0·0008677	9·9981305	63	4·7801622	98	4·7967493	133	4·8127263	168	4·8281362	55	9·9995995
21	0·0009111	9·9990870	64	4·7806450	99	4·7972141	134	4·8131742	169	4·8285685	56	9·9995613
22	0·0009544	9·9990435	65	4·7811272	100	4·7976784	135	4·8136216	170	4·8290005	57	9·9995237
23	0·0009977	9·9990000	66	4·7816090	101	4·7981421	136	4·8140688	171	4·8294319	58	9·9994866
24	0·0010411	9·9989564	67	4·7820902	102	4·7986054	137	4·8145153	172	4·8298629	59	9·9994502
25	0·0010844	9·9989129	68	4·7825709	103	4·7990681	138	4·8149614	173	4·8302937	60	9·9994144
26	0·0011277	9·9988694	69	4·7830511	104	4·7995303	139	4·8154070	174	4·8307238	63	9·9993115
27	0·0011710	9·9988258	70	4·7835306	105	4·7999921	140	4·8158523	175	4·8311536	66	9·9992161
28	0·0012143	9·9987823	71	4·7840098	106	4·8004533	141	4·8162970	176	4·8315830	69	9·9991293
29	0·0012576	9·9987387	72	4·7844883	107	4·8009142	142	4·8167413	177	4·8320119	75	9·9989852
30	0·0013009	9·9986952	73	4·7849664	108	4·8013744	143	4·8171852	178	4·8324404	81	9·9988854
31	0·0013442	9·9986516	74	4·7854438	109	4·8018343	144	4·8176285	179	4·8328686		

thermometer is ***highest*** at the ***lowest*** station, and the number in the right-hand column of B being taken when the thermometer is ***lowest*** at the ***lowest*** station; finally, making M = the middle term, then

$$M = \log. \beta - (\log. \beta' + B).$$

The whole computation reduces itself therefore to this simple form:

$$\text{Log. } x = A + C + \log. M.$$

Example.

Stations.	Date and time of observation.	Height of Barometer.	Thermometers. Attached.	Thermometers. Detached.	Difference of level.	Latitude.	Remarks.
	Aug. 7, 1841.						
River side ..	9 h. A.M.	29·98	63°	62°	3685 ft.	54° N.	
Trig. station	6 h P.M.	26·17	47	45			
			D = 16	S = 107			

$$M = \log. \beta - (\log. \beta' + B)$$

$B =$		0·0006943
$\text{Log. } \beta'$	$= 26·17$	1·4178037
$(\log. \beta' + B) =$		1·4184980
which taken from $\log. \beta$	$= 29·98$	1·4768316
leaves $M =$		0·0583336

$$\text{Log. } x = A + C + \log. M$$

$+ A =$	4·8009142
$+ C =$	9·9996381
$+ \log. M =$	$\bar{2}$·7659188
$\text{Log. } x =$	3·5664711

x = difference of altitude = 3685·3 feet.

Observations to determine the relative altitude of two stations should, if possible, be made simultaneously with

two instruments previously compared, as reliance cannot be had on the atmosphere remaining in the same state during the time otherwise required to pass from one station to the other. Several observations should be made at each station at agreed intervals of time, and the mean results taken to serve as terms of the comparison. When there is only one observer, he should take three or four observations at each station, passing from one to the other as quickly as possible. If many stations are visited during the day, the observer should endeavour to repeat, at the close of the day, observations at the first station, to enable him to judge whether any great change has taken place in the height and temperature of the barometer, as also in the temperature of the surrounding air, during the period occupied in the observations. He judges thereby of the degree of dependence to be placed on the observations; and if he have, as shown in the form of field-book given above, entered the time of each observation, he may correct the readings in the proportion of the total change indicated by the first and last reading, to the change due to each interval of time, assuming that the alteration between the first and last reading has been the result of a uniform progressive change between the first and last observations.

CHAPTER X.

MINING SURVEYS.

The value to man of all the metals as main instruments, by the aid of which he emerges from the savage state; and the influence of the mineral fuel of which Great Britain possesses such valuable accumulations, on the well-being of man and his advance in civilization, tend to rank the mining interests among those of which the unchecked prosperity is most important to society*. But of all speculative employments, mining is one of the most uncertain; and casual failures, caused too often by the want of precise knowledge of really accessible data, lead to the neglect and total abandonment of subterranean works, which might, with such information, have contributed in an eminent degree to the wealth of the proprietors, and the increased power of the community. All the resources of science ought, therefore, to be concentrated and brought to bear on mining operations, in order to lessen as much as possible the chances of disappointment. For many years, geology has lent its powerful aid to guide the bold adventurer in his subterraneous labours; but it has been too common in practice, hitherto, to neglect the construction of accurate surveys, which, although they may not lay claim to such paramount influence as the physical science we have named, are nevertheless primary elements of success.

A mining plan is the chronicle wherein are recorded all geological and mineralogical data connected with the mining district. It serves also as the guide by which

* See Dr. Buckland's *Bridgewater Treatise*, chapter xix.

new workings may be directed; and, if made with strict geometrical accuracy, it saves one heavy item of expense in all subterraneous works, namely, that arising from lost labour in driving false headings. It guards also against the occasional destruction of human life by unexpectedly opening into a gallery, or cutting into a protecting dyke, supposed by the indications of an inaccurate plan to have been at a greater distance.

The want of subterranean plans, constructed with as great a degree of geometrical precision as those representing the surface of the ground, has been long felt by scientific and practical men. With a very few exceptions, no correct or trustworthy records of subterranean works are preserved in any of the important mining districts of Great Britain, and so strong is the impression of too general neglect in this particular, that the highest authorities in mining engineering have recommended, "that in future leases of mines, the proprietors should introduce a clause to require the adventurers to keep sections and plans of all their workings." But, manifestly injurious as the general inexactness of mining plans is acknowledged to be, the error continues to prevail, that mining surveys can be performed with instruments incomparably less accurate, and with precautions much less stringent, than those which are now deemed indispensable to the perfect success of surveys on tho surface. It is further to be observed that surveys on the surface seldom present obstacles equal to thoso of subterraneous works, in many of which the difficulty of access, and the great irregularity and varied ramifications of the levels, demand all the care and skill which the experienced surveyor can command.

It may be said to be the universal practice at present to perform mining surveys by means of the mariner's or miner's compass for the observation of horizontal angles.

No strictly accurate surveys can be performed by such means, and the following are among the causes of error inhereut to the present system.

First. Angles cannot be measured nearer to the truth than from $\frac{1}{4}$ to $\frac{1}{2}$ a degree with the miner's compass, the diameter of which is rarely large enough to admit of a more minute subdivision of the circumference,—but especially because the mode of action of the needle does not admit of the application of a vernier to the circumference.

Secondly. The horizontal or azimuthal angles, indicated by the magnetic needle, cannot be read with precision, because the needle is seldom parallel to the plane of the instrument, but has either its north or south pole raised some distance from the plane. Unless then, in viewing the needle, the eye be kept exactly in the vertical plaue passing through its longitudinal axis, the index will not be seen projected upon its proper place on the circle; and as we evidently cannot, in reading the angle, always be certain that the eye is placed in the required position, we are frequently liable to refer the index to the wrong division, or incorrectly to appreciate its apparent or parallactic distance from the right division.

Thirdly. The diurnal variations of the needle are too important to be neglected, as they sometimes amount to $\frac{1}{2}$ a degree in an iuterval of twelve hours. The use of the compass necessitates, therefore, an inquiry into, and a record to be made of, the variations of the needle at different periods of each day's work, as well as on consecutive days. For in the very same locality, variations of 1° have been observed in an interval of fourteen days, and of $1\frac{3}{4}$° in an interval of six months*.

Lastly. The use of the common miner's compass is evidently inapplicable to mines of such minerals as are possessed of undoubted magnetic properties,—and inde-

* *Annales des Mines*, tome ix., 1836, page 88.

pendently even of these, the needle may be made to deflect through local attractions, among which might perhaps be included the electro-magnetic currents to which Mr. R. W. Fox is inclined to trace the existence of metallic veins. Above all, the powerful influence of an iron railroad, a most common requisite in all mines, has long been ascertained;—so great is its effect, that the needle, when held at the joint between two rails, will immediately place itself in a direction parallel to their length *. This last cause of error may probably be destroyed by using an improved compass needle, a late invention of Mr. James Ramsden, intended to obviate the derangement of the nativity of the needle, caused by the presence of the iron rails, and various metallic substances. The improved needle is thus described in No. 293 of the *Mining Journal*, for 1841. "It consists of three parts, viz.: two bars of steel, each 1½ inch long, and each having a north and south pole, divided in the middle by a brass bar, which separates the north pole of one bar from the south pole of the other by about 1½ inch. The affinity of one of these for the other is so strong that the polarity of the needle as a whole is maintained."

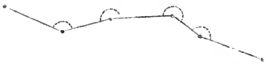
The above remarks amply testify the insufficiency of the miner's compass for the performance of accurate underground surveys, and to its inherent imperfections are added those of the instruments for measuring the angles of inclination.

The quadrantal or semicircular instruments used for this purpose are applied as follows:—A fine silken cord or brass wire is tightly stretched in the direction of the line to be measured: to this wire the diameter or straight side of the instrument is applied, and the angle of inclination is denoted on the divided arc by a plummet depending from its centre. If the distance be so long as to cause a sensible amount of deflection or curvature in the wire, the instru-

* *Annales des Mines*, tome ix., 1836, page 99.

ment is applied at two or three intermediate stations, in order to counteract the effect of the curvature by combining the several readings. The instrument thus acting without the aid of a telescope directed to a well-defined fixed point, can give only an approximation to the truth.

Finally, the common method of protracting the work on the plan, by causing each succeeding bearing to embrace



and confirm all the antecedent errors of observation and plotting, can lay no claim to accuracy.

This last description of errors is not, it must be observed, inherent to the principle of surveying with the compass; for the method of plotting by reference to three normal co-ordinates, as we shall presently explain, is equally applicable to the system of observation just described as to that which it is proposed to substitute for it, namely, observation with the theodolite. The substitution of the theodolite is recommended on the grounds of incomparably greater accuracy, and also of greater rapidity of execution, inasmuch as it avoids the cumbrous method of taking the angles of inclination as above described.

The common theodolite, in order to adapt it to mining observations, requires a slight modification in the construction of the stand, whereby it may be easily disengaged from the staff for the purpose of placing it on any support or bearing afforded by a locality too confined to admit of the common stand being used. The inferior parallel plate should have in its middle part, a hollow socket of several inches in depth, and slightly conical. This is intended to receive a brass projection attached to the stand which serves the same purpose as the screw of the staff head in the ordinary construction. To proceed with the survey

of a mine, if its adits or galleries be not too steep and irregular, 3 tripod stands are provided similar to those

used in common field work, but shorter, and having, instead of the common staff head, a flat broad head, (with 2 small spirit levels at right angles to each other,) on which can be screwed, when required, a brass pivot made slightly conical, so as to fit into the hollow socket of the lower parallel plate, which is made fast to it by a friction screw.

If the galleries be too steep and rough to admit of the steady adjustment of such tripods, wooden supports may be substituted of the annexed form, armed with spikes or

Side view.

Plan.

cramps to fasten them to the polling boards or shores on the sides of the galleries.

Lamps are used as the objects to be observed: they are made so as that the focus of the light shall be at the same elevation, when placed on one of the tripods or rests, as the axis of the theodolite when fixed in the same place. A hollow socket beneath the lamp, provided also with a friction screw, fits on the projecting brass stand of the tripods or rests.

To carry forward a survey with due correctness and expedition, the surveyor must have the assistance of two chainmen and two labourers to carry the instruments and fix the forward lamp. A lamp is placed at the starting point from which the chaining commences, and the surveyor goes forward with the theodolite, in the direction of the line to be surveyed, to the farthest point from which he can see the light of the lamp back. One of the tripod stands is then placed at that point to receive the theodolite, and the telescope is directed towards the light, the vernier being fixed at zero; the angle of inclination is also

noted. During the time occupied in this operation, one of the assistants has moved forward with a lamp and a tripod stand, which he fixes at the farthest point at which his light can be seen from the instrument. The forward angle is then read on the horizontal limb, and the angle of inclination having also been noted, the instrument is taken forward to the stand on which the second light was placed, and the lamp is itself removed to serve for a second forward station, while the first light is brought forward to the first position of the theodolite, and the observations are continued as before. The chainmen measure the distances at the same time that the instrument is moved forward by the surveyor, and he enters all the observations in the field-book according to the following form.

No. of hypotenuse.	Measured distances in feet.	Angles of inclination.	Readings of horizontal limb.		Horizontal angles.	Remarks.
			Back sight.	Fore sight.		
		° ′ ″	° ′	° ′	° ′	
1	133	depr. 2 30 0 elev. 4 29 30	0 0	189 16	189 16	
2	150	depr. 4 29 30 elev. 7 32 0	189 16	357 29	168 13	
3	302	depr. 7 31 30 elev. 15 20 0	357 29	178 53	181 24	
4	220	depr. 15 20 0 elev. 0 30 0	178 53	357 41	178 48	

The operation above described will be seen to be similar to that practised for road or town surveys, and known by the name of "traversing." However, the operation as conducted in subterraneous surveying, presents certain advantages not possessed by that usually carried on on the surface. First, the mode of observing with three fixed supports so contrived that the observed object and the axis of the instrument always occupy in succession the same position, prevents all inaccuracies from eccentricity of the instrument or the object staves. This eccentricity, in the common method of traversing on the surface,

forms a serious and practically unavoidable element of error, for the axis of the theodolite being fixed in position by means of a plummet suspended over a picket of a certain diameter, cannot with precision be placed truly in the axis marking the centre of the object-staff just removed. This defect would be entirely obviated by using, as recommended for subterraneous surveys, three tripod stands.

Secondly, the superior distinctness and precision afforded by a small bright light used as a point of sight in mining surveys, is well appreciated by all who have had the opportunity of comparing it with the intersection of the centre of a fixed vane placed across the staff, used as the object of sight in surveys on the surface.

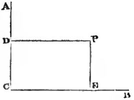
Lastly, in subterraneous surveying, the surveyor is protected from all unsteadiness in the instrument, consequent on the agitation of the atmosphere.

From these remarks, it may be inferred that, as "traversing" is a process mathematically correct in theory, although to a certain extent erroneous in practice, the mining surveyor has the means of approaching more nearly to geometrical precision, by following the practical directions given above.

The work may be plotted at once from the field-book, by means of a circular protractor, as described under the head of "traversing," page 145. That method, by referring all the angles to a common meridian, presents the advantage that a trifling error in the *direction* of one of the lines does not affect the *direction* of the lines succeeding. But it counteracts the tendency to accumulation of errors only as regards the *direction* of the lines, and not as regards their length, errors in which may continue to accumulate through each step of the plotting. By carrying out the principle to its full extent, and referring the lengths of the lines as well as their directions to common meridians or co-ordinates, this remaining liability to error in the plotting is obviated.

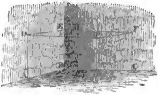
The position of a point in a *plane* can be determined by the lengths of two straight lines drawn from it parallel to, and terminated by, two lines given in position, and perpendicular to each other.

Thus, if the two straight lines, A C, C B, perpendicular to each other, and intersecting in C, be given in position, and P be any point in the same plane from which P D and P E are drawn parallel to B C and A C, if P D or C E, and P E be given, it is evident that the position of P would be determined. It is in this manner that offsets from a main or station-line determine the position of side-objects in a survey, or offsets from a datum line determine the section of a line of country in levelling.



Again, the position of a point in *space* can be determined by the lengths of three straight lines, formed by the mutual intersection of three planes at right angles to each other.

Thus, to determine the position in space of the point P, it is first referred to a horizontal plane A B C given in position, by a perpendicular P P′ drawn to that plane. The point P′ is then determined, as shown above, by two perpendiculars, P′ A, P′ C, drawn to the lines A B, B C, given in position. This is evidently the same operation as referring the point P to the three planes, A B C, A B E, C B E, perpendicular to each other, for the lines P′ A, P′ C, are evidently equal to the lines P D and P F respectively. The lines drawn from the point P to the three planes fixed in position are called co-ordinates. These three rectangular planes, by a reference to which the position of any point can be deter-

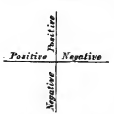


mined, may be compared to the floor and two of the side walls of a room*. Perpendiculars drawn from the point to the walls and floor would intersect them in points called their projections. This is the method universally employed by engineers and architects to convey a representation of the different parts of a proposed edifice,—the projections on the three planes being denominated *plans*, *elevations*, and *sections*. The plan is the projection of the parts of the edifice on the horizontal plane, the elevation and section are the projections of the parts of the edifice on two vertical planes at right angles to each other.

The terms of altitude, latitude, and longitude, are analogous to the three lines or co-ordinates drawn to three planes fixed in position.

It is proposed, then, to apply this universal method of determining the position of points in space to the plotting of mining surveys. For that purpose the three planes are supposed to intersect in the starting point of the survey, one of them being horizontal, the others vertical and perpendicular to each other. The measured lengths, together with the angles of inclination, determine the position of each point with respect to the horizontal plane; in other words, its altitude. This co-ordinate altitude is said to be positive when it lies above the horizontal plane; negative when it extends below it.

The measured lengths with the horizontal angles determine the position of each point with reference to the vertical



planes, one of which may be called the plane of the meridian, the other the parallel of the latitude. When a co-ordinate longitude lies to the right of the meridian it is positive, when it lies to the left it is negative. When a co-ordinate latitude lies above, or, as it were, to the north of the normal parallel

* Lacroix, *Complément des Elémens de Géometrie.*

of latitude, it is positive; when below it, or, as it were, to the south of it, it is negative.

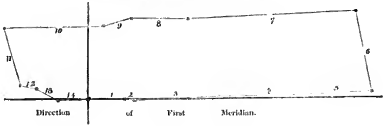
The solution of right-angled triangles, a process quickly performed by means of logarithmic tables, determines the position of all the points from the data given in the field-book. The steps in the process are best disposed according to the form given in the next page, in which the distances of each station from the three planes are registered independently of each other, and can be plotted in the same manner as a section. The additional time occupied in the calculations is, in some measure, saved in the plotting, which is much more expeditious when performed by means of two rectangular co-ordinates than by the use of the protractor.

When new galleries have to be conducted in connexion with, or having a reference to, old workings, it will be more accurate to set out their directions from data obtained from the table of registered lengths than to measure them from the plan.

Underground surveys require to be referred to, and connected with surveys on the surface. When the connexion can be made by adits, it is effected by traversing out of the adit to the surface: when it must be accomplished through a narrow shaft, the bearing of the first line, used in the underground survey, with the magnetic meridian is ascertained with the greatest care, and in order to detect the existence of any local cause of attraction, many observations should be made under different circumstances, and at different parts of the line of which the bearing is required. As soon as this has been satisfactorily determined, a line with the same bearing is set out at the mouth of the shaft, the surveyor using the same precautions to guard against local or accidental deflexion of the magnetic needle. The direction of the first line being thus marked on the surface, the other lines which have been traversed underground are set out, and marked by pickets on the surface, from the data given in the field-book.

No. of hypotenuse.	Angles of inclination. Elevation.	Depression.	Measured distances.	Horizontal angles.	Angles of direction with reference to plane of the meridian. +	−	Logarithms of measured distances	Logarithms of cosines of angles of inclination.	Logarithms of sines of angles of inclination.	Logarithms of sines of angles of direction.	Logarithms of cosines of angles of direction.
	° ′	° ′		° ′	° ′	° ′					
1		0 38	15·60		0 ·0		1·1931246	9·9999735	8·0435009	0·	10·
				189 18							
2		5 9	5·55		9 16		0·7442930	9·9982433	8·9520996	9·2069059	9·9942950
				168 13							
3		4 18	36·70			2 31	1·5646661	9·9987947	8·8715646	8·6425634	9·9995809
				181 24							
4		1 33	36·70			1 7	1·5646661	9·9998411	8·4321561	8·2897734	9·9999175
				178 48							
5	0 5		30·60			2 19	1·4857214	9·9999995	7·1626960	8·6066226	9·9996449
				82 34							
6	0 30		36·50			99 45 80 15 sup.	1·5622929	9·9999835	7·9408419	9·9936813	9·2287839
				97 21							
7	0 16		73·60		177 36 2 24 sup.		1·8668778	9·9999953	7·6678445	8·6219616	9·9996189
				182 39							
8	0 38		25·45			179 45 0 15 sup.	1·4056878	9·9999735	8·0435009	7·6398160	9·9999959
				165 25							
9	8 22		14·00		165 40 14 20 sup.		1·1461280	9·9973132	9·0448954	9·3936852	9·9862683
				192 58							
10	4 36		34·05		178 38 1 22 sup.		1·5321171	9·9985968	8·9041685	8·3774968	9·9998764
				96 21							
11	0 38		25·80		94 59 85 1 sup.		1·4116197	9·9999735	8·0435009	9·9983553	8·9388496
				94 32							
12	0 37		6·95		9 31		0·8419848	9·9999748	8·0319195	9·2183635	9·9939815
				203 1							
13		3 26	10·10		32 32		1·0043214	9·9992198	8·7773334	9·7306129	9·9258681
				143 45							
14		1 59	14·70			3 43	1·1673173	9·9997398	8·5391863	8·8117264	9·9990856

Plotted from the data given in the above Table.



Logarithms of measured distances reduced to horizontal base.	Logarithms of the distances of each station from three planes passing through each station			Distances obtained from logarithms in preceding columns.						Distances of each station from principal planes passing through first station.					
	Log. distances from			Distances from						Distances from					
Hypotenuse × cosine of angle of inclination.	Horizontal plane = hyp. × sin. of angle of inclination.	Meridian plane = horiz. dist. × sine of angle of direction.	Latitude plane = horiz. dist. × cosine of angle of direction.	Horiz. plane.		Plane of meridian.		Plane of latitude.		Horiz. plane.		Plane of meridian.		Plane of parallel.	
				+	−	+	−	+	−	+	−	+	−	+	−
1·1930961	$\bar{1}$·2360255	0·	1·1930961		0·17			15·60			0·17			15·60	
0·7425363	$\bar{1}$·6073926	$\bar{1}$·9494422	0·7368313		0·50	0·89		5·45			0·67	0·89		21·05	
1·5634606	0·4362307	0·2069242	1·5630417		2·73		1·61	36·56			3·40		0·72	57·61	
1·5645072	$\bar{1}$·9908232	$\bar{1}$·8542906	1·5644247		0·99		0·71	36·68			4·39		1·43	94·29	
1·4857209	$\bar{2}$·6484174	0·0923435	1·4853638	0·04			1·24	30·57			4·35		2·67	124·86	
1·5622764	$\bar{1}$·5031348	1·5559577	0·7910603	0·32			35·97		6·18		4·03		38·64	118·68	
1·8668731	$\bar{1}$·5347223	0·4888347	1·8664920	0·34		3·08			73·70		3·69		35·56	44·98	
1·4056613	$\bar{1}$·4491887	$\bar{1}$·0454773	1·4056372	0·28			0·11		25·45		3·41		35·67	19·53	
1·1434412	0·1910234	0·5371264	1·1297075	1·55		3·44			13·48		1·86		32·23	6·05	
1·5307159	0·4362856	$\bar{1}$·9082147	1·5305923	2·73		0·81			33·93	0·87			31·42		37·88
1·4115932	$\bar{1}$·4551206	1·4099485	0·3504428	0·29		23·70			2·24	1·16			5·72		30·12
0·8419596	$\bar{2}$·6739043	0·0603231	0·8359411	0·07		1·15		6·85		1·23			4·57		23·27
1·0035412	$\bar{1}$·7816548	0·7341541	0·9294093		0·60	5·42		8·50		0·63		0·65			14·77
1·1670571	$\bar{1}$·7063036	$\bar{1}$·9787835	1·1661427		0·51		0·95	14·66		0·12			0·10		0·11

In this series of observations a polygon has been described, the last measurement closing the work by returning to the station of departure. This example is taken from an excellent Essay on Subterraneous Surveying, published by M. Combes, in the ninth volume of the *Annales des Mines*, 3rd series; in which the application of this method of referring the data to three co-ordinates, which was first recommended by M. D'Aubuisson, is strongly advocated.

CHAPTER XI.

OF LATITUDE AND LONGITUDE.

The Sextant.

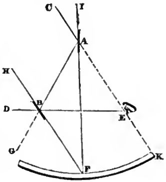
When it is required to measure the angular distance between two objects, the observer being himself on an unstable basis, (at sea for example,) the theodolite cannot be used for the purpose, because the principle of its construction, and the nature of its adjustments, demand that it should rest on a firm support. If, moreover, it be required to measure with perfect accuracy, at any specified moment of time, the angular distance between two objects in motion, this cannot be accomplished by the theodolite, because the telescope must be directed in succession from one object to the other, and a certain amount of time must necessarily elapse between the first and second sight.

The sextant (a modification of Halley's quadrant, so called from its reputed inventor, though the merit of the invention seems due to Newton*,) obviates these difficulties, as it may be used when held simply in the hand, and gives, by a single sight or observation, the angular distance between two objects. Its principle and construction are as follows:—

Principle of the Sextant. Let A and B be two mirrors moveable on axes parallel to each other, the second mirror B being half silvered to admit of the passage of rays of light through half its area.

* Sir J. F. W. Herschel's *Astronomy*, and Hutton's *Mathematical Dictionary*.

Let a ray of light, proceeding from the object C, be reflected from the mirror A, and after a second reflexion from the half-silvered glass B, enter the eye at E; also let a second object D be seen by direct vision through the half-silvered glass B, required the angle subtended at E by the objects C and D.



Produce the plane of the mirrors until they shall intersect in F, the angle $A F B = \frac{1}{2}$ angle $C E D$. For, producing $A B$ to G, we have

$$G B E = B A E + A E B, \text{ (Euc. I. 16.)}$$

But because the angle of incidence is equal to the angle of reflexion

$$H B A = F B E = G B F, \text{ (Euc. I. 15,)}$$

$$\text{therefore } G B E = 2 G B F = B A E + A E B;$$

$$\text{but } 2 G B F = 2 B A F + 2 A F B, \text{ (Euc. I. 16,)}$$

$$\text{therefore } B A E + A E B = 2 B A F + 2 A F B.$$

But because

$$C A I = B A F = F A E,$$

$$B A E = 2 B A F,$$

taking equals from equals, we have

$$A E B \text{ (or } C E D) = 2 A F B^*.$$

* If the mirrors be placed at such an angle that the image of the object C is reflected to the eye at E; then we have

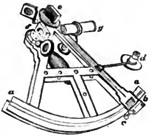
$$C E B = E B A + E A B, \text{ but}$$
$$E B A = 180^\circ - 2 A B F, \text{ and}$$
$$E A B = 180^\circ - 2 B A F, \text{ therefore}$$
$$C E B = 360^\circ - (2 A B F + 2 B A F), \text{ but}$$
$$2 A F B = 360^\circ - (2 A B F + 2 B A F), \text{ therefore}$$
$$C E B = 2 A F B.$$

If the two mirrors be parallel to each other, and the angle formed by their planes equal therefore to zero, then a

distant object seen by direct vision, and its reflected image, will appear to coincide; the parallax of the two mirrors, or their separation, not being sufficiently great to render the deviation from parallelism between the direct and reflected rays sensible.

If to the mirror A (see fig. page 221), moveable round an axis perpendicular to the plane A F B G, an index A F be attached, passing along a graduated arc F K, and if the mirror B be fixed with its plane, parallel to C K and to the axis of the mirror A, then the angle F A K = angle A F B = ½ angle C E D. If the arc D K, therefore equal to one-sixth of the circumference or 60°, be divided as if it were equal to 120°, the number of degrees marked, as included in the arc F K, will be equal to the angle C E D.

Construction of the Sextant. The annexed figure represents a sextant so framed as not to be liable to bend. The



a Graduated arc.
b Vernier.
c Tangent-screw.
d Microscope.
e Mirror, or index-glass.
f Half-silvered glass.
g Eye-tube.

arc is generally graduated to 10′ of a degree, which are subdivided by the vernier into 10″, a space equal to 59 divisions on the arc being subdivided on the vernier into 60 parts. An arm carrying the vernier or index moves

round an axis placed at the centre of the circle, of which the graduated limb forms an arc. Over this axis or centre of motion, and attached to the arm, a mirror is fixed perpendicular to the plane of the instrument, so that a movement given to the index is communicated to the mirror. The index is clamped and adjusted by the usual clamp and tangent screws. A second glass, half silvered to admit of direct and reflected vision, is attached to the frame, nearly opposite the first mirror, and with its plane perpendicular to the plane of the instrument. The zero of graduation on the limb is placed so that the vernier shall indicate zero when the two mirrors are parallel to each other.

Objects are observed with this instrument, either through a plain tube, or through a telescope. It is better for the learner to use the plain tube, owing to the increased difficulty of bringing the objects into the field of view when the telescope is employed.

Dark glasses, of different depths of shade and colour, are attached to the instrument, to be used when the sun is observed, so as to moderate the intensity of the light.

*Adjustments of the Sextant**. "The requisite adjustments are the following: the index and horizon-glasses must be perpendicular to the plane of the instrument, and their planes parallel to each other when the index division of the vernier is at 0° on the arc, and the optical axis of the telescope (when used) must be parallel to the plane of the instrument."

"*Adjustment of the Index-glass.* Move the index forward to about the middle of the limb, then, holding the instrument horizontally with the divided limb from the observer, and the index-glass to the eye, look obliquely down the glass, so as to see the circular arc, by direct view and by reflection, in the glass at the same time; and if they appear

* SIMMS on *Mathematical Instruments.*

as one continued arc of a circle, the index-glass is in adjustment. If it requires correcting, the arc will appear broken where the reflected and direct parts of the limb meet. This in a well-made instrument is seldom the case, unless the sextant has been exposed to rough treatment. As the glass is in the first instance set right by the maker, and firmly fixed in its place, its position is not liable to alter, therefore no direct means are supplied for its adjustment.

" *To examine the Horizon-glass, and set it perpendicular to the Plane of the Sextant.* The position of this glass is known to be right, when by a sweep with the index, the reflected image of any object passes exactly over or covers its image as seen directly; and any error is easily rectified by turning the small screw at the lower end of the frame of the glass.

" *To examine the Parallelism of the Planes of the two Glasses, when the Index is set to Zero.* This is easily ascertained; for, after setting the zero on the index to zero on the limb, if you direct your view to some distant object, the sun for instance, you will see that the two images (one seen by direct vision through the unsilvered part of the horizon-glass, and the other reflected from the silvered part) coincide or appear as one, if the glasses are correctly parallel to each other; but if the two images do not coincide, the quantity of their deviation constitutes what is called the index error. The effect of this error on an angle measured by the instrument is exactly equal to the error itself: therefore, in modern instruments, there are seldom any means applied for its correction, it being considered preferable to determine its amount previous to observing or immediately after, and apply it with its proper sign to each observation. The amount of the index error may be found in the following manner: clamp the index at about 30′ to the left of zero, and looking towards the sun, the two images will appear either nearly in contact or overlapping each other; then

perfect the contact, by moving the tangent-screw, and call the minutes and seconds denoted by the vernier, the reading *on* the arc. Next place the index about the same quantity to the right of zero, or on the arc of excess, and make the contact of the two images perfect as before, and call the minutes and seconds on the arc of excess the reading *off* the arc; and half the difference of these numbers is the index error; additive when the reading on the arc of excess is greater than that on the limb, and subtractive when the contrary is the case.

Example.

Reading on the arc . .		31′ 56″
„ off the arc . .		31 22
Difference		0 34
Index error . . .	=	− 0 17

In this case the reading on the arc being greater than that on the arc of excess, the index error, = − 17″, must be subtracted from all observations taken with the instrument, until it be found, by a similar process, that the index error has altered."

"*To make the Line of Collimation of the Telescope parallel to the Plane of the Sextant.* This is known to be correct, when the sun and moon, having a distance of 90° or more, are brought into contact just at the wire of the telescope which is nearest the plane of the sextant, fixing the index, and altering the position of the instrument to make the objects appear on the other wire; if the contact still remains perfect, the axis of the telescope is in proper adjustment; if not, it must be altered by moving the two screws which fasten, to the up-and-down piece, the collar into which the telescope screws. This adjustment is not very liable to be deranged."

Of the Use of the Sextant. The large sextant is rarely required for observations on land of terrestrial or of celestial bodies; but, for purposes of navigation or for maritime surveying it is of essential importance. Instruments, in fact, constructed on this principle are the only instruments capable of being used on ship-board for determining altitudes, or measuring the angular distances of objects.

The instrument is held lightly in the right hand, and moved until its face is in the plane of the two objects, the angular distance of which is required. When altitudes, therefore, are to be observed, the instrument is held in a vertical plane; when horizontal or oblique angles are to be measured, it is held in a horizontal or oblique plane.

When the altitude of an object, the sun, for instance, is to be observed at sea, where no level or artificial horizon can be used, "the observer, having the sea horizon before him, must turn down one or more of the dark glasses, according to the brilliancy of the object; and directing his sight to that part of the horizon immediately beneath the sun, and holding the instrument vertically, he must with the left hand lightly slide the index forward, until the image of the sun, reflected from the index-glass, appears in contact with the horizon, seen through the unsilvered part of the horizon-glass. Then clamp it firm, and gently turn the tangent-screw, to make the contact of the upper or lower limb of the sun and the horizon perfect, when it will appear a tangent to his circular disc. To the angle read off apply the index error, and then add or subtract the sun's semidiameter, as given in the *Nautical Almanac*, according as the lower or upper limb is observed, to obtain the apparent altitude of the sun's centre. Before we can use this observation for determining the time, the latitude, &c., it must be further corrected for refraction and parallax, to obtain the true altitude, subtracting the former and adding the latter; and when the sea horizon is employed, a quantity must also be subtracted for the dip, which is

unnecessary when the altitude is taken by means of an artificial horizon.

"Tables for obtaining the above corrections may be found in Mr. Bailey's *Astronomical Tables*, &c., in the *Requisite Tables*, or in any modern work on navigation.

"*Example.*

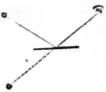
Obs. alt. of the sun's lower limb		=	61° 13′ 5″
Index error		=	− 17
Apparent altitude		=	61 12 48·0
Sun's semidiameter		=	+ 15 46·9
Sun's parallax		=	+ 0 4·0
			61 28 38·9
Refraction	− 34·4″		
Dip of the horizon, for an elevation of 18 feet	− 4 3·0	=	− 4 37·4
True altitude of the sun's centre		=	61 24 1·5."

When a "lunar distance," *i. e.*, the distance between the sun and moon, or between the moon and a fixed star or planet, is required, the instrument is held in the plane of the two objects, the fainter object being observed by direct, the brighter by reflected vision.

The angular distance between two terrestrial objects is measured in the same manner.

The inconvenience of measuring, with the sextant, angles between terrestrial objects, whose horizontal distance is that which is generally required, is, that the angles, being measured in planes parallel to the plane in which the two objects are situated, have to be reduced to their horizontal value, as explained in page 115.

When the altitude of a celestial object is to be taken on



land with the sextant, an artificial horizon is used. Of these there are various constructions, all of which aim at presenting a reflecting plane parallel to the natural horizon, from which the

rays of the celestial object may be reflected to an eye placed in the direction of the rays of reflexion. The angle measured in such a case is double the altitude of the object above the true horizon.

Among the various fluids used for the purpose of presenting such a reflecting surface mercury has been found most useful. But during the observations its surface must be protected from agitation by the external air; for this

purpose a roof-shaped cover is placed over the trough in which the mercury is contained, two plates of glass being fixed in the sides. These plates of glass should have perfectly parallel faces to avoid irregular refraction; but as this cannot be ensured exactly, two observations ought always to be made with the roof in reversed positions, in order to correct any error occasioned by undue refraction.

Of Latitude and Longitude.

The latitude of a place on the earth's surface is its angular distance from the equator, and it is equal to the altitude of the elevated pole above the horizon. This angular distance or latitude can be determined by observing the altitude of the pole, or the greatest altitude of a celestial body whose declination at the time of the observation is known: the declination of a celestial body being its distance from the equinoctial, or the complement of its distance from the pole.

Deeming it of great importance, that the principles upon which the solution of the various problems connected with longitude should be understood, we are induced to insert the following eminently clear exposition of those principles from Sir J. F. W. Herschell's *Astronomy*, pages 133, *et seq.*

"To determine the latitude of a station is easy. It is otherwise with its longitude, whose exact determination is a matter of more difficulty. The reason is this: we are obliged in both cases to resort for their determination to marks external to the earth, *i. e.*, to the heavenly bodies; but to observers situated at stations on the same *meridian*, *i. e.*, differing in latitude, the heavens present different aspects at *all* moments; to observers situated on the same *parallel*, *i. e.*, differing only in longitude, the heavens present the same aspects. In the former case there *is*, in the latter there is *not*, anything in the appearance of the heavens, watched through a whole diurnal rotation, which indicates a difference of locality in the observer.

"But no two observers, at different points of the earth's surface, can have at the same instant the same celestial hemisphere visible. Suppose, to fix our ideas, an observer situated at a given point of the equator, and that at the moment when he noticed some bright star to be in his zenith, and therefore on his meridian, he should be suddenly transported, in an instant of time, round one quarter of the globe in a westerly direction, it is evident that he will no longer have the same star vertically above him: it will now appear to him to be just rising, and he will have to wait six hours before it again comes to his zenith, *i. e.*, before the earth's rotation from west to east carries him back again to the line joining the star and the earth's centre, from which he set out.

"The difference of the cases, then, may be thus stated, so as to afford a key to the astronomical solution of the problem of the longitude. In the case of stations differing only in latitude, the same star comes to the meridian at the same *time*, but at different *altitudes*. In that of stations differing only in longitude, it comes to the meridian at the same *altitude*, but at different *times*. Supposing, then, that an observer is in possession of any means by which he can certainly ascertain the *time* of a known star's transit across

his meridian, he knows his longitude; or, if he knows the difference between its times of transit across his meridian and across that of any other station, he knows their difference of longitude. For instance, if the same star pass the meridian of a place A at a certain moment, and that of B exactly one hour of sidereal time, or one twenty-fourth part of the earth's diurnal period, later, then the difference of longitudes between A and B is one hour of time or 15°, and B is so much west of A.

"In order to a perfectly clear understanding of the principle on which the problem of finding the longitude by astronomical observations is resolved, the reader must learn to distinguish between time, in the abstract, as common to the whole universe, and therefore reckoned from an epoch independent of local situation, and *local time*, which reckons, at each particular place, from an epoch or initial instant, determined by local convenience. Of local reckoning we have instances in every sidereal clock in an observatory, and in every town clock for common use. The sidereal clock is regulated by observing the meridian passages of the more conspicuous and well known stars, and this operation is called getting the *local time*.

"Suppose, now, two observers, at distant stations A and B, each independently of the other, to set and regulate his clock or chronometer to the true sidereal time of his station. It is evident that if one of these chronometers could be taken up without deranging its going, and set down by the side of the other, they would be found on comparison to differ by the exact difference of their local epochs; that is, by the time occupied by any star, in passing from the meridian of A to that of B; or in other words, by their difference of longitude, expressed in sidereal hours, minutes, and seconds.

"Were chronometers perfect, nothing more complete and convenient than this mode of ascertaining differences of longitude could be desired. An observer, provided with

such an instrument, and with a portable transit, or some equivalent method of determining the local time at any given station, might, by journeying from place to place, and observing the meridian passage of stars at each, (taking care not to alter his chronometer, or let it run down,) ascertain their difference of longitude with any required precision.

"The chronometer, however, though greatly improved by the skill of modern artists, is not yet sufficiently perfect to be relied on implicitly. Observers have therefore sought to resort to other means of communicating from one station to another a knowledge of its local time. The simplest and most accurate method by which this can be accomplished is by telegraphic or other signals, such as the flash of gunpowder, the explosion of a rocket, the sudden extinction of a bright light, or any other which admits of no mistake, and can be seen from one station to the other. The moment of the signal being made is noted by each observer by his respective watch set to local time, consequently, when the observers communicate their observations of the signal to each other, the difference of their local time, and therefore of their longitudes, becomes known. But circumstances seldom admit of the use of these artificial signals; natural ones have therefore been employed as their substitute; and the eclipses of Jupiter's satellites being visible at once over a whole terrestrial hemisphere, afford, in addition to their universality, the great advantage that the time of their happening at any fixed station, such as Greenwich, can be predicted with great certainty. An observer, therefore, at any other station wherever, who shall have observed one or more of these eclipses, and ascertained his local time, instead of waiting for a communication with Greenwich, to inform him at what moment the eclipse took place there, may use the *predicted Greenwich time* instead, and thence, at once, and on the spot, determine his longitude. The predicted Greenwich time is always published five years in

advance in the *Nautical Almanac*. The nature of this observation is, however, such that it cannot be made at sea, so that, however useful to the geographer, it is of no advantage to navigation. "Moreover, the returns of the eclipses are of only occasional occurrence; and in their intervals, and when cut off from all communication with any fixed station, it is indispensable to possess some means of determining longitudes on which the geographer and navigator can implicitly rely for a knowledge of their positions. Such a method is afforded by

"*Lunar Observations.*

"If there were in the heavens a clock furnished with a dial-plate and hands, which always marked Greenwich time, the longitude of any station would be at once determined, so soon as the *local time* was known. Now, the offices of the dial-plate and hands of a clock are these:—the former carries a set of marks upon it, whose position is known; the latter, by passing over and among these marks, informs us, by the place it holds with respect to them, what it is o'clock, or what time has elapsed since a certain moment when it stood at one particular spot.

"In a clock, the marks on the dial-plate are uniformly distributed all around the circumference of a circle, whose centre is that on which the hands revolve with a uniform motion. But it is clear that we should, with equal certainty, though with much more trouble, tell what o'clock it were, if the marks on the dial-plate were *un*equally distributed,—if the hands were eccentric, and their motion not uniform,—provided we knew, 1st, the exact intervals round the circle at which the hour and minute marks were placed; which would be the case if we had them all registered in a table, from the results of previous careful measurement;—2ndly, if we knew the exact amount and direction of eccentricity of the centre of motion of the hands;—and, 3rdly,

if we were fully acquainted with all the mechanism which put the hands in motion, so as to be able to say at every instant what were their velocity of movement, and so as to be able to calculate, without fear of error, *how much time* should correspond *to so much angular* movement.

"The visible surface of the starry heavens is the dial-plate of our clock, the stars are the fixed marks distributed around its circuit, the moon is the moveable hand, whose position among them can at any moment when it is visible be exactly measured by the help of a sextant, just as we might measure the place of our clock-hand among the marks on its dial-plate with a pair of compasses, and thence, from the known and calculated laws of its motion, deduce the time.

"Such a clock would, no doubt, be considered a very bad one; but, if it were our *only* one, and if incalculable interests were at stake on a perfect knowledge of time, we should justly regard it as most precious, and think no pains ill bestowed in studying the laws of its movements, or in facilitating the means of reading it correctly. Such, in the parallel we are drawing, is the lunar theory, whose object is to reduce to regularity the indications of this strangely-irregular going clock, to enable us to predict, long before-hand, and with absolute certainty, whereabouts among the stars, at every hour, minute, and second, in every day of every year, in Greenwich local time, the moon would be seen from the earth's centre, and *will* be seen from every accessible point of its surface; and such is the *lunar method* of longitudes. The moon's apparent angular distances from all those principal and conspicuous stars which lie in its course, as seen from the earth's centre, are computed and tabulated with the utmost care and precision in the *Nautical Almanack**. No sooner does an observer, in any part of

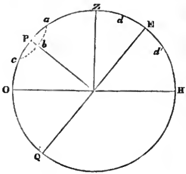
* "The *Nautical Almanac* is published annually, by order of the Lords Commissioners of the Admiralty, generally four years forward;

the globe, at sea or on land, measure its actual distance from any one of those *standard stars*, (whose places in the heavens have been ascertained for the purpose with the most anxious solicitude,) than he has, in fact, performed that comparison of his local time with the local times of every observatory in the world, which enables him to ascertain his difference of longitude from one or all of them."

Having prepared the reader by the above simple explanations to understand the general principles on which the determination of latitude and longitude depends, we now proceed to illustrate their practical application.

To determine the Latitude of a Place.

First, by the mean altitude of a circumpolar star observed at the time of its upper and lower culmination. This method is the simplest, as it requires no correction for the declination of the star observed,—the observation is made with a transit instrument or theodolite, carefully adjusted in the plane of the meridian, the readings of the vertical arc being noted at the time of the upper and lower transits of the star. These readings are then corrected for



in it are entered the sun's longitude, right ascension, declination; the planets' longitudes, latitudes, times of passing the meridian; the times of solar and lunar eclipses, together with those of Jupiter's satellites; the distance of the moon from the sun and certain fixed stars, at the beginning of every third hour; and in general the times when any remarkable appearances take place, being all computed for Greenwich time.

atmospheric refraction from tables prepared for the purpose, and the mean reading gives the latitude of the place. For, if $a\,b\,c$ be the path of the star about the pole P, Z the zenith, and H O the horizon: then is a O the altitude of the star upon the meridian when above the pole, and c O the same when below the pole; hence because $a\,P = c\,P$ when both are corrected for refraction, therefore $\frac{a\,O + c\,O}{2} = o\,P = E\,Z$; hence the height of the pole O P, is equal to E Z the latitude of Z, *i.e.*, its angular distance from the equator.

A second method is by a single observation of the meridian altitude of the sun, or a star whose declination is known. Thus the altitude H d or H d' of the celestial body d or d' being observed, its zenith distance, or co-altitude, is known. Then to the zenith distance add the declination d E, (as given in the *Nautical Almanac*,) when the star and place of observation are on the same side of the equator, or subtract the declination d' E when they are on different sides; and the sum or difference will be the latitude E Z reqnired. In this case, as in the former, the observed altitudes must be corrected for refraction; for parallax caused by the distance of the observer from the earth's centre; and for the semi-diameter of the sun when it is the object observed.

Example 1. Ou August 11, 1841, the double meridional altitude of the sun's lower limb was observed with a sextant to be 104° 27′ 45″; required the latitude of the place of observation, the observer being north of the sun:—

Double altitude . . .				2) 104°	27′	45″
				52	13	52
⊙'s semidiameter	= + 16′	0″	+			
Refraction .	= − 0	44			15	21
Parallax . .	= + 0	5				
True altitude				52	29	13

Co-altitude, or zenith distance .	37° 30′ 47″
Aug. 11, 1841; declination north +	15 18 00
Latitude	52 48 47

When the natural horizon is used instead of an artificial horizon for an observation made with the sextant, an additional correction for the dip of the horizon is required: this correction is also obtained from tables prepared for the purpose, and which are published in Bailey's *Astronomical Tables*, or in any modern work on navigation.

Example 2. On September 21, 1829, in longitude 60° E. the meridian altitude of the sun's lower limb was 56° 26′, the observer being south of the sun, and the height of his eye 26 feet above the surface of the sea,—required the latitude of the place of observation:—

Altitude of ⊙	56° 26′ 00″
⊙'s semidiameter	+ 0 16 00
	56 42 00
Dip of the horizon	− 0 4 52
Apparent altitude ⊙	56 37 08
Refraction − 38·4″ } correction . Parallax + 4·4 }	− 0 34 00
True altitude ⊙	56 03 08
Co-altitude, or zenith distance .	33 56 52
Sept. 21, 1829; decl. north − 0° 43′ Correction for longitude + 4	− 0 39 00
Latitude	33 17 52

To determine the Longitude of a Place.

Of the various methods which astronomers have devised for determining the longitude of a place, we shall only describe the method of determining it by "lunar observa-

tion," a method which possesses the advantage of being easily applied at sea, and does not involve complex calculations. For additional information on this subject we would refer to PEARSON'S *Practical Astronomy*, RIDDLE'S *Navigation*, NORIE'S *Navigation*, &c.

To find the true angular distance of the moon from a star or the sun, it is necessary that the altitudes of the moon and that of the other object, whether a star or the sun, be measured in order to correct the observed angular distance from the effects of parallax and refraction. For the moon is always seen lower than her true place, because, owing to her vicinity to the earth, the apparent depression caused by parallax is a greater quantity than the apparent elevation caused by refraction: the sun is always seen higher than its true place, its great distance rendering parallax of less effect than refraction. These apparent changes from the true positions cause the true distance to be almost always greater or less than the observed distance.

For greater accuracy the three observations, *i. e.*, the angular distance and the two altitudes, should be taken simultaneously; but if they are taken in succession by one observer, he is to bestow especial care in measuring the lunar distance, taking the altitudes as rapidly as possible. He proceeds in the following order, taking first, the two altitudes with the time of each observation; secondly, the lunar distance repeated several times with the time of each observation, from whence a mean of the times and distances is deduced; lastly, the altitudes in reverse order. The altitudes are then reduced to the mean time of the lunar distance by the following proportion. As the difference of times between the observations is to the difference of the corresponding altitudes, so is the difference between the time at which the first altitude was taken and the mean time, to a fourth number which, added or subtracted from the first altitude, according as it is increasing or

diminishing, will give the altitude reduced to the mean time*.

The obtaining of the true distance, called "clearing the lunar distance," is a problem in spherical trigonometry. Of such vital importance at sea is its correct solution, that the most eminent astronomers have turned their attention to the subject with the view to simplify it. Tables, the results of their labours, are given in all works on navigation, with directions for their use; by their means, an operation otherwise laborious is much expedited, and placed within the reach of all who are moderately versed in arithmetic.

Having the true distance, the longitude is obtained as follows†:—

Rule "1. Among the true distances of the moon's centre from the sun or fixed stars which are set down in the *Nautical Almanac*, find those two distances on the given day that are next less and greater than the true distance found by the observation, which place under it: take the difference between the true distance and the first of these two distances, also the difference between the two distances; subtract the proportional logarithm of the second difference from the proportional logarithm of the first difference, and the remainder will be the proportional logarithm of a portion of time, which, added to the time that the first of the two distances, taken from the almanac, was computed for, the sum will be the apparent time (*i. e.*, the time derived from the sun by observing its transit over the meridian) at Greenwich.

"2. Take the difference between the apparent time at Greenwich and the apparent time at the ship; convert it into degrees and minutes, and it will give the true longitude; east if the time at the place of observation be greater, but west if the time be less, than the time at Greenwich reckoned from the same apparent noon."

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* NORIE's *Navigation*, page 224. † Ibid., page 238.

Example. Apparent time of observation $3^h\ 56^m\ 17^s$

True lunar distance	79° 16′ 25″				
Distance at noon from *Nautical Almanac*	79 14 17	first dif.	0° 2′ 8″	pro. log. 1·9262	
Distance at 3, from do.	80 40 7	second dif.	1 25 50	pro. log. 0·3216	
				pro. log. 1·6046	

Proportional part	0^h	4^m	28^s	
Time of first distance	0	0	0	
Apparent time at Greenwich	0	4	28	
Do. at place of observation	3	56	17	
Dif.	3	51	49	= 57° 57′ 15″ E. lon.

Meridian Line.

The method of obtaining a meridian line by observations of the sun, given in page 118, would be perfectly accurate, if the sun moved constantly in the same parallel. However, his advance in the ecliptic, or the change in his declination during the interval elapsed between the first and last observations, requires a correction on the mean results of the observations, varying according to the season of the year. The required correction is greatest at the time of the equinoxes, as the change in the sun's declination is then the most rapid. The middle point of the horizontal arcs, as obtained by equal altitudes, is to the west of the true meridian when the sun is advancing towards the elevated pole, and to the east of the true meridian when he is receding from the elevated pole.

To apply the correction to two observations of equal altitude, the time of each observation must be noted:

Let T = time of first observation

T′ = time of second observation

D = sun's declination at the time of the first observation } as obtained from the *Nautical Almanac.*

D′ = do. second observation

then the

$$\text{Correction} = \tfrac{1}{2}(D - D') \text{ sec. lat. cosec. } \frac{(T - T')}{2}.$$

Practical Rule. To the log. of half the change of declination during the interval between the observations, add the log. secant of the latitude, and the log. cosecant of half the interval of time between the observations converted into space; the sum − 20 will be the log. of the correction in seconds of space.

*Example**. "The readings of the horizontal limb at equal altitudes of the sun were 130° 10′ 15″ and 32° 36′ 15″, therefore the middle point or reading of the approximate meridian was 81° 23′ 15″. The interval of time between the observations was 5 hours, the half of which converted into space = 37° 30′. The sun's hourly change of declination = 56·77″, therefore the change for half the interval = 141·92″ (approaching the north pole). The latitude of the place = 51° 28′ 39″, required the correction to be applied to the middle point to obtain the direction of the true meridian.

½ (D − D′) = 141·92″	log.	2·1520436
Lat. = 51° 28′ 39″	sec.	0·2056388
½ (T − T′) = 37° 30″	cosec.	0·2155529
Correction 374·31″	log.	2·5732353
= 6′ 14·31″		

Reading of the middle point	81° 23′ 15″
Correction	− 0 6 14
Reading of the instrument when set to the true meridian	81 17 1

* Simms on *Mathematical Instruments*, page 97.

When the meridian is deduced from equal altitudes of a circumpolar star no correction is required.

When it is desired to obtain the meridian line by a single observation of a circumpolar star, the observer bisects the star when it is near its greatest eastern or western elongation, and follows it by moving the tangent screw until it appears to remain stationary. The reading of the horizontal limb is then registered, and the azimuth of the star or its distance from the meridian computed by the following rule:—

"From the log. sine of the polar distance of the star, subtract the log. cosine of the latitude; the remainder will be the log. sine of the azimuth required."

Adding or subtracting this azimuthal angle (as the case may be) from the reading of the horizontal limb, the sum or the remainder will give the angle on the limb, to which the vernier is to be adjusted, in order that the telescope may point to the true meridian.

The pole star, and other stars whose angular distance from the pole is small, are well suited for this purpose, as, owing to their slow apparent motion when at their greatest elongation, they may be bisected with great nicety.

Most instructive and interesting details on the determination of meridian lines, as also on the mode of conducting the measurement of arcs of a meridian, and of determining geodesically the latitudes and longitudes of stations, will be found in the second volume of the *Trigonometrical Survey of England and Wales.*

CHAPTER XII.

ON MARITIME SURVEYING.

Maritime Surveying has for its object the determining (for the purpose of representation on hydrographical plans or charts) of coasts, harbours, inlets, rocks, shallows, soundings, and whatever particulars may serve to direct the mariner on his voyage, or point out the dangers to be avoided.

Practical Directions. Observations for the construction of a chart are made with reference to fixed points on shore. The relative positions of those points are ascertained, either with great precision, or with a degree of approximate accuracy proportionate to the extent of detailed information to be given on the chart. When perfect accuracy is aimed at, many stations on shore are in the first instance fixed in position by means of a trigonometrical survey, executed according to the methods previously described, and in which the accuracy of the work is to be tested as usual by the measurement of one or more bases of verification. The stations in the triangulation being selected with reference to the ultimate end in view, will be chosen so as to determine the position of remarkable headlands, beacons, lighthouses, and other objects of primary importance to the mariner. With these data, whatever extent of coast may be embraced by the projected hydrographical work, each series of operations at sea will be confined to spaces comparatively limited, and the whole will consist of numerous detailed charts correctly joined and harmonized by means of the triangulation on shore. A description, therefore, of the mode of operation adopted for the marine survey of a single harbour or limited sea-reach, will apply equally to

the system adopted in the performance of a continuous survey embracing an extensive line of coast.

The triangulation on shore is generally performed in the first instance, but it may proceed simultaneously with the maritime survey, taking care that it shall be kept somewhat in advance of the latter.

Tide Gauges. The triangulation being supposed completed, the first step for the maritime operations consists in the selection of localities suited to the erection of tide-gauges, divided into feet and tenths, to be fixed in a vertical position. The zero point of each gauge is to be referred to a fixed permanent bench mark by means of the spirit-level, in order that the gauge may be refitted in its original position, should it be displaced by the violence of the sea or some other cause.

After a series of observations, these gauges serve, in the first instance, to give the lowest point of the lowest tide at full and change of the moon,—and to the level of this lowest point the depths of all soundings are referred. The gauges, in addition, give for every day and portion of each day, on which soundings are made, the amount of rise and fall of the tide,—and, by means of these latter observations, all registered soundings are reduced to the lowest level. The necessity for this is obvious, inasmuch as it would be impossible to take the soundings even of a limited area at the precise time of low water.

By taking advantage of a quay or other local circumstance, the observations relating to the higher stages of the tide may frequently be made from the shore, though they, perhaps, cannot be carried down to low water. A second gauge must then be provided and placed further out to seaward, so that when the tide shall be on the point of leaving the first, the observer may proceed to the second. In such a case, the relative altitude of the zero division of the first gauge, as compared with some division of the second, must be carefully ascertained with the spirit-level, and, each gauge being denoted by a distinctive letter, proper entries must be

made in the field-book to record the time of changing from one gauge to the other. In most cases, however, a careful selection will enable the observer to find a suitable locality in which the base of the tide-gauge will not be left dry by the retreating tide.

A trustworthy assistant, provided with a well-regulated watch, is to be stationed at each tide-gauge for the purpose of registering the height of the tide at regular intervals of time, usually every quarter of an hour. At each tide-gauge station, a meridian line should be marked (see pages 118 and 240), in order that the tide registrar may regulate his watch by the course of the sun. If at any time it be required to alter the time of the watch, both the date and the amount of such alteration are to be entered in the field-book.

Time of High Water at Full and Change. It is important to take advantage of the opportunity offered by the tide-gauge to note with great precision the time of high-water at full and change of the moon, as this information is always required on the chart. While on this subject, we cannot do better than to add the following "suggestions on the observations of the heights and times of the tides," as given by Professor Whewell.

First, as to time: "The time used in tide observations may be mean or apparent time, but it should always be noted which is employed, and by what means obtained."

"The *establishment* of any place, or the time of high-water at full and change of the moon, should always be noted in the field-book, not as is commonly done," at such an hour of the day, "but as being so many hours after the moon's transit, the time of which is easily known from the tables." "If the tide be observed according to mean time, and the time of the moon's transit be determined according to apparent time, it will be necessary to apply the equation of time to the interval."

Secondly, as to time of high-water: "The instant to be

registered is of course that when the surface of the water is highest; but if the water be perfectly still, it changes very slowly when near the highest point, and appears to be stationary for some moments. To avoid the difficulty produced by this circumstance, some observers have registered, not the time when the water is highest, but two instants of equal height before and after the greatest; and the time of greatest height is supposed to bisect this interval."

To obviate the effect of waves in rendering the surface uncertain, the following apparatus may be used Let a pipe be fixed upright by the side of the gauge, in such a situation that at low-tide time the water shall reach its lower part. The bottom of the pipe must be stopped, and a number of small holes, about a ¼ inch in diameter, be made in or near the bottom. A float nearly filling the pipe is to be placed in it, and to carry a light upright rod, divided into feet and decimals, which are to be read off by means of an index or mark fastened to the top of the tube. The apertures in the bottom of the tube will allow the float to rise and fall with the general surface, without any sensible loss of time; while the smallness of those apertures will prevent the oscillations of the waves from affecting the inside of the tube.

No precise directions can be given as to the proper number of stations for gauges: this must be determined from the information obtained from pilots or fishermen on the coast as to the variations in the amount of rise and fall of the tide at different spots. As a general rule, it may be observed that in all narrow channels, and especially in rivers where obstacles cause greater accumulations of water and consequent exceptional irregularities in the change of the level of the tides, a greater number of gauges are to be used than on an open sea-board.

The annexed table may be used as a form for the registry of the tide-tables required for surveying operations.

Form of Tide-Table.

At Station No. 1. *Entrance into* ________ *River.*

July , 184 .

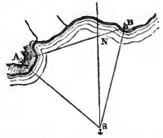
Month. Day.	Moon's transit.			Moon's age at noon.	Remarks.
	h. m.	h. m.	feet.	days.	
21		11 15 A.M.	7·7		Watch set at apparent time, by meridian observation of the sun.
		30	8·1		
		45	8·4		
		12 0 noon	8·9	26·4	Watch 1 minute slow.
		15	9·2		
		30	9·4		Light wind, S.S.E.
		45	9·5		
	9 11 mean time.	1 0	9·6		
		15	9·5		
		30	9·4		
		45	9·1		
		2 0	8·7		
		15	8·2		
		30	7·9		

Operations at Sea.

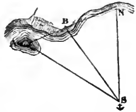
If there be not on the shore permanent well-defined stations, such as churches, towers, lighthouses, or other beacons fixed by triangulation, the surveyor erects the necessary signals at the vertices of the triangles. Those which are, when viewed from the sea, projected on the ground behind them should be painted white; those which are projected against the sky, or on a sandy beach, should be painted black or red. These preliminaries arranged, the observer is prepared to commence his operations at sea, having secured the assistance of an able pilot, and of men skilful in the use of the lead.

Three methods may be used for determining, by reference to fixed points on shore, the locality of any station at sea, such as a rock, shoal, reef, &c.

The first consists in observing, by means of an azimuthal compass, the bearing of two or more points on shore, whereby the position of the observer is determined when the position and bearing of the points on shore are given with respect to each other. For, let A and B be two objects on shore, fixed in position with reference to one another and to their bearing with the meridian; and let S be the position of the observer, from which the angles A S N and B S N, formed by the objects A and B with the meridian, are observed.

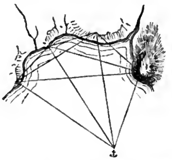


In the triangle A S N, the angles at S and N are given, consequently the angle at A is known; in the same manner the angle at B in the triangle B S N is known. Then in the triangle A B S the side A B and the adjacent angles A and B being given, the point S may be found.



This method gives at best but a loose approximation, because the angles cannot be determined by the compass nearer than within about 1 or 2 degrees of the truth: the only recommendation in its favour is its great rapidity and facility of execution. When resorted to, it is advisable to employ two compasses, one at a height of 2 or 3 feet above the other, and use the arithmetical mean of the two readings.

The second method consists in observing at the same time, by means of preconcerted signals, from two or more

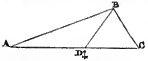


stations on shore, the bearing of the observer at sea with some fixed objects. Theoretically, this method is the most accurate, but practically, it is found that even well concerted signals cannot always ensure simultaneous observation. As the times of observation must moreover be registered at all the stations as well as at sea, a single error in the series arising from unseen signals leads to constant misapprehension, and can scarcely be rectified by a subsequent comparison of the different field-books, if the series embrace many observations. Independently of these objections, others present themselves in the form of a greater consumption of time, and a necessity for an increased number of experienced observers.

The third method consists in measuring from the boat, or vessel, or rock, by means of the sextant, the angles subtended by three or more objects on shore, the positions of which are given,—from these data the position of the observer is determined.

Theorem *.

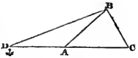
The mutual distance of three remote objects being given, with the angles which they subtend at a station in the same plane, to find the relative place of that station.



Let the three points A, B, and C, and the angles A D B and B D C which they form at a fourth point D, be given: to determine the position of D.

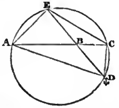
* LESLIE'S *Trigonometry*, page 385, *et seq.*

First. Suppose the station D to be situated in the direction of two of the objects, A and C.

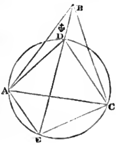


All the sides A B, A C, and B C of the triangle A B C being given, the angle B A C is found, and in the triangle A B D, the side A B with the angles at A and D being given, the side A D is found, and consequently the position of the point D is determined.

Secondly. Suppose the three objects A, B, C to lie in the same direction.



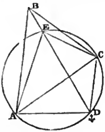
Describe a circle about the extreme objects A, C, and the station D; join D A, D B, and D C; produce D B to meet the circumference in E, and join A E and C E. In the triangle A E C, the side A C is given, and the angles EAC and E C A, being (Euclid III. 21) equal to C D E and A D E, are consequently given; wherefore the side A E is found. The triangle A E B, having thus the sides A E and A B, and their contained angle E A B or B D C given, the angle A B E and its supplement A B D are found. Lastly, in the triangle A B D, the angles A B D and A D B, with the side A B, are given; whence BD is found. But since the angle A B D and the distance B D are assigned, the position of the station D is evidently determined.



Thirdly. Let the three objects form a triangle, and the station D be either within or without it.

Through D, and the points A and C, describe a circle: draw B D cutting the circumference in E, and join E A, and C E.

1. In the triangle A E C, the side A C, and the angles A C E and C A E, which are (Euclid III. 21) equal to A D B or its supplement, and to B D C or its supplement, being given, the side A E is found.



2. All the sides of the triangle A B C being given, the angle C A B is found.

3. In the triangle B A E, the sides A B and A E are given, and the contained angle E A B, (being either the difference or the sum of C A E and C A B,) is also given, whence the angle A B E or A B D is found.

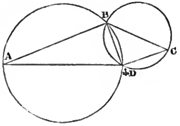
4. In the triangle D A B, the side A B and the angles A B D and A D B being given, the side A D or B D is found, and consequently the position of the point D, with respect to A and B, is determined. By a like process the relative position of D and C is deduced; or C D may be calculated from the sides A C, A D, and the angle A D C, which are given in the triangle C A D.

It is obvious that the calculation will fail, if the points B and E should happen to coincide. In fact the circle then passing through B, any point D whatever in the opposite arc A D C will answer the conditions required, since the angles A D B, and D B C, being now in the same segment, must remain unaltered.

This third case, in which the three objects form a triangle, involves the conditions under which the problem has in general to be solved, the first case in which two of the objects, and the second in which the three objects are in a line, occurring but rarely. The reader will, however, doubtless have been impressed with the extremely laborious nature of the solution which this case involves, demanding no less than four separate trigonometrical calculations before the required answer is obtained. It would evidently, therefore, be a most tedious process, and one but little suited

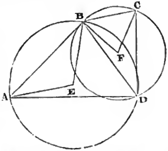
to practical purposes; other means have therefore been devised of solving the problem which are better suited to practice.

The first of these consists in a geometrical construction. Let A, B, and C be the three stations, and D the position of the observer, at which the angles A D B, B D C have been measured.



On A B (Euclid III. 33) describe a segment containing an angle equal to that subtended by the objects A and B, and on B C describe another segment B D C, containing an angle equal to that subtended by the objects B and C; the point D, where the two circumferences intersect, will evidently mark the station required. Should the two circles have the same centre, their circumferences must obviously coincide, and therefore every point in the containing arc will answer the conditions required, in which case the problem becomes indeterminate.

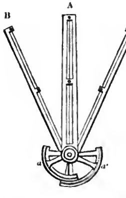
Example. Let the three objects on shore, A, B, C, be fixed in position; and let the angle subtended at D by A B be equal to 50°, and the angle subtended by B C be equal to 40°; to find the point D by construction. Subtract double the angle A D B from 180°, and take half the remainder, equal 40°. Lay off this angle at A and B, the two lines forming the angles with A B will meet in E, the centre of a circle passing through A, B, D (Euc. III. 20). Again, subtract



double the angle B D C from 180°, and take half the remainder, equal to 50°. Lay off this angle at B and C; the two lines forming the angles with B C will meet in F, the centre of a circle passing through B, C, and D. The point D, where the two circles intersect, marks the station required.

But this process, although much simpler in point of construction than that previously explained, would yet be exceedingly tedious where a great number of stations had to be determined. To simplify the construction, an instrument, called the station-pointer, has been invented: it affords means of laying down the work with great rapidity, and with sufficient accuracy for all practical purposes. The following is a description of the instrument.

The *station-pointer* is formed by three limbs or rulers, A, B, C, which revolve round a common centre, in such a manner that B and C may be set to form any angles with A.



"The middle ruler is double, and has a fine wire stretched along its opening; the other rulers have likewise a fine wire stretched from end to end, and so adjusted by the little projecting pieces which carry them, that all the three wires tend to the centre of the instrument, where they would meet if produced. Through the centre is an opening sufficiently large to admit a steel pricker." The middle limb carries, at the extremities of two arms, the verniers *a* and *a'*. Two arcs, of about 100° each, are connected with the limbs B and C, in such a position, that when the instrument is closed the verniers *a* and *a'* mark zero; and when the limbs are opened, the angles they respectively form with A are marked by the verniers on

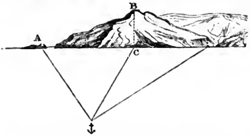
their corresponding arcs. The angles subtended by three stations on shore, at the place of the observer at sea, are the measures of the readings to which the verniers are to be set; and when properly fixed, the instrument is laid on the plan, and moved till the three wires pass through the three stations: the centre of the instrument then occupies the relative place of the observer, and a dot marked by the steel pricker determines the point D on the plan.

It is evident, from this description, that in the absence of the station-pointer, a graduated circle marked on any transparent paper, or on a plate of glass, may be used instead, by drawing on the upper surface of such a circle lines diverging from the centre at the given angles; the circle being moved until these radii pass through the stations, the centre of the circle will give the point required.

The demonstration given above (page 250) proves that, with the exception of the case in which the observer is in the circumference of the circle passing through the three stations, the measure of two angles is sufficient to determine his position. In practice, however, as many angles as possible ought to be observed to the surrounding stations, in order to obtain greater accuracy; and in no case ought the observer to rest satisfied with the measure of two angles only, unless necessity compels him to adopt this extreme limit.

The angles are observed with the sextant or reflecting circle, and are consequently measured in the plane of the objects. If this plane be inclined to the horizon, and a result rigorously accurate be sought, the angles of elevation of each station above the horizon should at the same time be observed, to afford data for reducing the hypotenusal to the horizontal angle. But this reduction may be neglected in all cases where the difference of elevation between the objects does not exceed 2° or 3°, and when the observed angle is larger than 20° or 25°; for the reduction to the horizon would in such cases deal with quantities more

minute than the amount of error to which the measures of all angles observed at an unstable station are liable.



When the difference of elevation between the objects is considerable, an ideal vertical line (see sketch) may be drawn from the higher object downwards to an elevation corresponding to that of the lower object, and this, with some experience and correctness of eye, will give results sufficiently near to the truth.

With the sextant no telescope should be used, because the objects are more quickly brought into the field of the mirrors by the unassisted eye, and rapidity of execution is most important in the observations.

Metallic reflectors should be used instead of the glass mirrors, as sea water, to the effect of which the sextants are constantly exposed in such observations, rapidly destroys the silvering. Another reason for preferring the metallic reflectors is on account of the indistinctness caused by the object being reflected from both surfaces of the transparent mirror, which indistinctness is very much increased when a telescope is used.

When there is only one observer engaged in the operation, he should, when measuring the angles subtended by objects on shore, have at least three sextants ready at his hand, so as to measure with different instruments the separate angles in rapid succession, without losing the time that would otherwise be necessary to enter the reading of the sextant after each observation. For on unstable stations,

such as boats or vessels, the angles should be taken as nearly as possible at the same instant.

The field-books should be kept with an indelible pencil, the mark of which is not liable to be effaced by the washing of sea-water. The form of field-book to be used in registering the angles is given below. In the first column is entered the index error of the sextant; in the second, the precise moment of commencing the observations, with remarks as to the "status" of the boat or vessel, whether at anchor or otherwise; in the third column are entered the angles. The single letter opposite the bracket marks the first station on shore or station of departure, and a note is made stating whether the other stations are to the right or left of the first.

July, 1841. *Bay of* __________.

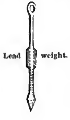
Index error. Sextant.		Observed Angles.	Remarks.
		To the right,	
Sextant 1..0′ 10″	h. m. 8 10	A { I .... 26° 18′	
,, 2..0 20	boat at	B 68 54	
,, 3..1 0	anchor.	C 89 42	
		To the left,	
Errors, as before	8 25	C { A 58 28	
	steadied by	B 24 32	
	the oars.	E 82 4	This angle doubtful.

Sounding Lines.

Sounding lines should be made of strong pliable cord, known as "lead line," divided into feet by different coloured rags, or other marks. The lead, fastened at one extremity, is shaped like the frustum of a cone, with the base hollowed out to hold some grease, to which the sand or mud at the bottom of the water may adhere, serving thus to show the probable nature of the anchorage. Lines are used differing in length and strength, and leads differing in weight according to the depth of the water in which the casts are made. The lines, especially when new, must be occasionally compared during a day's work with a standard measure always at hand, as they are liable to great and sudden changes. It is almost needless to observe that, in open waters, an experienced leadsman must be employed, whose reading of the depths should nevertheless be frequently checked. When the soundings are deep, the boat's way must be stopped at each cast, in order that the depth may be measured in a vertical direction.

On shallows or reefs near the surface, and generally in all anchoring grounds of small depth, where accuracy is consequently of the utmost importance, sounding rods, divided into feet, and weighted at the extremities, may with advantage be substituted to obtain greater correctness.

The grease let into the hollow base of the lead, or the "arming," shows the nature of the surface of the bottom; but before we pronounce upon the quality of an anchorage, we should likewise know, if possible, the nature of the material for some depth under the immediate surface. This object is accomplished by means of a lance-shaped pike, of a length and weight proportionate to the depth to which it is desired to penetrate beneath the surface. The part



below the lead is indented in the same manner as a rasp, and the indentations on its surface bring up specimens of the deposit or formation traversed by the instrument, thus indicating in some degree its nature. If the pike be impeded in its progress by rock, its bent or broken point, when brought up to the surface, gives evidence of the fact. Additional value may, by such means, be given to the soundings; and in the field-book, at the entries of the nature of the bottom, a mark should be made to distinguish the data obtained by this instrument from those obtained by the common lead. (See form of field-book, page 261.)

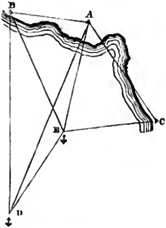
Survey of Shallows, Reefs, Sunken Rocks, &c.

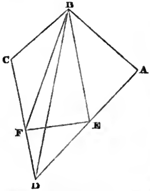
As it is of importance that the position of shallows, reefs, sunken rocks, &c., should be determined with the utmost possible accuracy, the observer should, whenever practicable without actual danger, cast anchor while making his observations, in order to ensure greater accuracy in the measurement of his angles, choosing for his stations on shore those which will subtend the largest angles at the place of the boat. Several remarkable points being thus determined at anchor, others may be fixed by means of two angles taken with rapidity while the boat is steadied by the oars. Sails ought rarely to be used when observations are made in these cases, as it is impossible under such circumstances to steady the boat. Moreover, the sails obstruct the sight in the measurement of the angles.

When it is required to determine a shoal or reef, &c., so far out to sea that only two objects on shore are visible, an assistant boat is moored temporarily between the distant observer and the coast, in such a place that one additional station can be seen from it. At a given signal, angles are observed from the assistant boat to the three objects on shore and to the distant boat; and from the

latter, angles are at the same signal measured to the two stations on shore and to the assistant boat.

Thus, in the annexed figure, let D be a station on the distant reef from which the two elevated stations A and B can be seen, let E be the position of the assistant boat from which the three stations A, B, and C can be seen; then, at the same moment of time, the angles C E A, A E B, B E D are observed from E, and the angles E D A and A D B from D,—and by their means the position of D is determined. For the point E is fixed in position by means of the observed angles C E A, A E B,—and it becomes therefore a fixed station with reference to D, from which two angles are observed to three stations fixed in position *.





* This question becomes somewhat more difficult of solution if only one angle can be observed from E. It is enunciated thus by Leslie:

"The mutual distances of three remote objects, two of which only are seen at once from the same station, being given, with the angles observed at two stations in the same plane, and the intermediate direction of these stations being also given,—to find their relative places.

"Suppose the three points A, B, and C, are given with the angles A E B and B F C, and likewise the angles A E F and E F C; to find the relative situation of the stations E and F.

If breakers or currents denoting danger are observed in a certain state of the tide, and it be impossible at the time to anchor over them, or to fix a buoy to mark their locality, their position should be determined approximately by intersections of prominent objects on shore, so disposed as to guide the observer to the spot in a more favourable state of the tide, when a perfect calm may leave no trace whereby the dangerous pass can be recognised.

A sketch or profile of the coast should be made before each series of important observations; and the stations should be defined and referred to on the sketch. These profiles are useful, not only in assisting the observer to recognise the coast when constructing his chart, but also in presenting to mariners the appearance of headlands and other striking points.

"Produce A E and C F to meet in D, and join B D; the angle E D F being equal to A E F + C F E − 180° (Euc. I. 13, and Cor. 32), is given. Now in the triangle E B F,

Sin. B F E : sin. E B F : : E B : E F,

and in the triangle E D F,

Sin. E D F : sin. D F E : : E F : E D;

wherefore, (Euc. V.,)

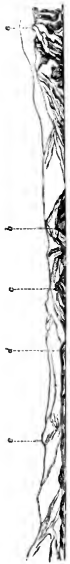
Sin. BFE × sin. E D F : sin. E B F × sin. D F E : : E B : ED,

and consequently the ratio of E B to E D is found. Again, the angle B E D, being the supplement of A E B, is given, and

Sin. B F E × sin. E D F : sin. E B F × sin. D F E : : E B : E D,
: : R : tan. b,

and R : tan. (45° − b) : : cot. ½ B E D : − cot. (½ B E D + E B D) or cot. (180° − ½ B E D − E B D), whence the angle E D B is given. The angles which all the three objects, A, B, C, subtend at the point D are therefore all given, and hence the position of D is determined by the preceding proposition. But B D being found, the several distances B E, E D, and B F, F D, are thence obtained, and consequently the position of each of the stations E and F is determined."

As this operation is somewhat too laborious for practice, in such exceptional cases as this, when only two stations could be seen, the azimuth compass might be used, as by its means the position of a station is determined with only two fixed objects in sight.—Leslie's *Trigonometry*, page 389.



On the chart directions are given for sailing or working into harbour, such as "Lighthouses in one, S. E. ½ S., lead over the bar, and up the channel to within two cables of the buoys." Such and similar information is to be entered on the authority of trustworthy pilots and others well acquainted with the locality.

While standing off and on to detect shoals or changes of level, a certain fixed direction must be followed and entered in the field-book, and whenever the direction changes, the point thus formed becomes a station from which angles are to be taken to the fixed objects on shore. Sonndings are to be taken at each of these stations, and also in passing from one station to another. The soundings taken at the stations are entered in a column opposite the observed angles, and any intermediato soundings are to be entered between them. When constructing the chart, the intermediate sonndings are to be distributed at equal distances along the line between each pair of stations, the time of taking the soundings being noted only at the stations where angles are observed.

The annexed form of field-book will illustrate the mode of operation.

Index error. Sextant.	Hours and minutes.	Angles observed to objects on shore.	Soundings. Feet.	Remarks. July, 1841.
20″				The watch being compared with that of the tide-registrar, the first sounding is taken near Station A.
	8 10		3; 5, rock; 7; 9; 12, rock; 14, sand.	Proceeding in a line from A to F.
	8 19 steadied by the oars.	To the right, A { I ..26° 18′ B ..68 54 C ..89 42 }	15, sand.	
			19; 24; 29.	Continuing in same direction.
	8 25	To the right, H past C { I ..58 28 B ..24 32 A ..82 4 }	31, sand and mud.	
			40; 45; 49; 50.	Ditto.
	8 35	To the left, I { A ..22 32 B ..28 48 E ..28 48 }	52, mud.	
			48; 39; 36, mud; 32, mud.	Direction changed towards C.
	8 50 at anchor.	To the left, C { B .... 38 36 I .... 63 24 A ....107 30 To the right, D .... 26 18 E .... 67 36 }	29, sand;—rock by the sounding-pike.	
	9 0 weigh anchor.		25; 20, sand; 19; 20, sand and shells.	Continuing in same direction.
	9 10	To the right, C { D ..30 4 E ..81 16 F ..95 20 }	29, shells and mud.	
			39; 30, sand; 21; 17, sand; 8, sand.	Change direction, and proceed S. by compass, in order to cross the bar which appears to connect the rock E with the shore.
	9 25	To the left, E { D ..21 13 C ..49 1 F ..77 55 }	6, sand.	
			8; 13, sand; 21; 27; 35, sand.	Continuing in same direction.

Reduction of Soundings.

The reduction of soundings consists in deducting from the depths, as registered in the field-book, proportionate quantities varying with the time, in order that all the depths may be referred to the lowest level of the tide. These quantities are obtained from the data supplied by the tide-registrar, and are arranged according to the annexed form for each day on which soundings and observations have been made.

Name of tide-registrar.	Date. 1841.	Time.			Deductions.	Time.			Deductions.
		h. m		h. m.	feet.	h. m.		h. m.	feet.
A. B.	July.	8 0	to	8 15	5	3 25	to	3 55	9
		8 15		8 35	4	3 55		4 30	10
		8 35		9 5	3	4 30		5 0	11
		9 5		11 0	2	5 0		5 25	12
		11 0		11 35	1	5 25		5 45	12
		11 35		0 10	2	&c.		&c.	&c.
		0 10		0 30	3				
		0 30		1 10	4				
		1 10		1 40	5				
		1 40		2 10	6				
		2 10		2 45	7				
		2 45		3 25	8				

For example, if the sounding over the highest part of a sand-bar gave a depth of 27 feet at the time that the annexed table or the tide-gauge indicated an elevation of 11 feet above the lowest tide, the sounding marked on the chart would be $27 - 11 = 16$ feet. If the depth over the shoal were registered at only 6 feet at the same time that the annexed table or tide-gauge indicated an elevation of

11 feet above low-water, then the true level of that part of the shoal at full and change of the moon would be $6-11$, indicating that it would be left high and dry at an elevation of 5 feet above the lowest tide.

The final entry in ink of the soundings is made in fathoms and quarter fathoms on all English charts: no decimals ought to be used, because the non-observance of the decimal point or its accidental omission by the engraver, a case of no rare occurrence, would lead to most disastrous consequences which would not necessarily follow from a mistake made in the entry of a fraction.

Maritime Surveying without the aid of Triangulation on Shore.

We now proceed to describe a method of constructing charts embracing a great extent of coast, second in point of accuracy to that which has for its basis a regular system of triangulation. This triangulation demands a considerable expenditure of time and money, and, of course, requires a free access to the country. It cannot, therefore, be put in practice where the country is in the occupation of an enemy, nor in cases where a considerable outlay of money is inexpedient. The following method, free from those drawbacks, is capable of giving results remarkable for their comparative accuracy: its principle or characteristic is this, that instead of commencing, as in triangulation, with the measurement of a short base from which a network of triangles is to be spread over a great extent of country, the first step in this process consists in using for the basis of the operations a very long base, say of 40 or 50 miles, and filling in the intermediate details by working from the whole to part, thus subdividing, at each intermediate step, any error that may exist in the original base, instead of accumulating error upon error, as must be the consequence were the

contrary system to be adopted, without the checks which a trigonometrical operation affords. It may be observed here, that in triangulation on shore, the accumulation of errors is prevented by the extreme care and precaution which we have described as necessary when treating on that subject; and that errors, if any have crept in, are detected by the measurement of the bases of verification.

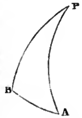
A judicious choice of the primary stations, or the stations determining the extremities of the great base, is important; but of course no precise directions can be given which shall be applicable to all circumstances of locality and climate. In general it may be observed, that the stations should be elevated peaks or headlands easily recognised; they must be at a considerable distance from each other (say 40 or 50 miles); but still the distance must not be so great as to prevent several intermediate points from being seen from both stations by the use of good instruments. Such intermediate stations are to be selected along the whole line of coast to be surveyed.

The locality of the first station being selected, its latitude and longitude are to be obtained by careful astronomical observations made from it (see pages 234 and 238); and in addition, if the observations be made by day, the azimuthal bearings of as many important points as can be seen between the first and second primary stations are to be taken. But should any hostile feeling of the inhabitants prevent the observations being made by day, these last objects cannot, of course, be observed, but greater time may be bestowed on the astronomical observations; the latitude being obtained by observations of the pole star, reduced by means of the tables given in the *Nautical Almanac;* and the longitude by observation of some of the stars usually selected for that purpose.

Without stopping at any intermediate points, similar observations are to be made consecutively at all the *primary* stations along the whole line of coast to be embraced in

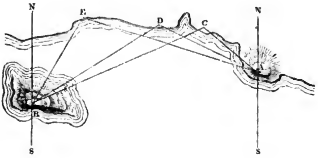
the survey. This continuity in the astronomical observations affords the important advantage, that the relative positions of the stations are determined while the rate of the chronometer remains unchanged; and, in sailing back to the starting point to commence the work in detail, a thorough acquaintance with the leading features of the coast is obtained, and an occasional check may be made on the previous observations.

The distance between any two of the primary stations whose latitudes and longitudes are thus determined, is obtained as follows:—



In the annexed figure, let P represent the pole of the earth, and A and B the two stations. The longitudes being known, the angle P, or their difference of longitude, is given. The sides P A and P B are also given, being the respective co-latitudes of the two stations. We have, therefore, in the spherical triangle A B P two sides, and the contained angle from which we get the length of the opposite side A B.

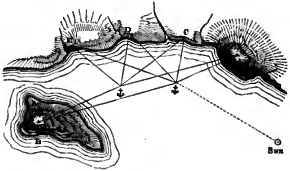
It is evident that the line joining any two of these stations is the arc of a great circle on the earth's surface: it must, therefore, be reduced to its chord, which is equal to twice the sine of half the arc (see page 120). This reduc-



tion having been attended to, several intermediate points will also have been determined, if the observations at the primary stations have been made by day, as shown in the last figure, where A and B are the primary stations, from which the azimuthal bearings of the stations C, D, E, &c., have been observed.

But if the observations have been made at night, when no intersections could be obtained, intermediate points are to be fixed in position by forming two or more temporary stations of the vessel anchored at convenient distances from A and B, and about 8 or 10 miles from the shore.

The watches having been regulated with great care during the observations made at night, the position of the vessel is determined by observing the angles subtended between the sun and the primary stations, and noting the exact time of the observations.



The time gives the sun's azimuth, and from it is deduced the azimuth of the two primary stations from the vessel. The intermediate stations C, D, E, &c., are obtained by the intersection of their lines of direction as observed at two or more of such stations of the vessel. Then a secondary series of points on shore nearer to each other is determined from the vessel at a distance of 2 or 3 miles

from the coast. Lastly, when these are protracted, the diagram is ready to receive the topographical details of the coast, and the soundings, the first being marked by sketching, the second according to the method already described. This process is repeated between the several primary stations, and the entire chart being then joined, represents an extensive district, the details of which have been obtained by proceeding from the whole to part, as before recommended.

Maritime Surveying under Sail.

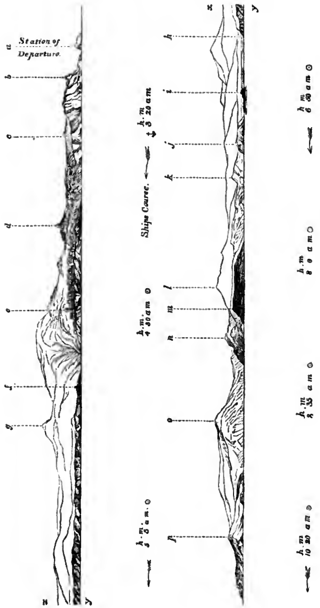
Third, in point of accuracy, but nevertheless highly useful, and sufficiently correct to be of great assistance to the mariner, is the survey made of a coast while sailing along it. It is, in fact, in this manner, that nearly all original maritime surveys of new colonies, or newly explored lands, have been made. This process differs in some of its details from those we have before described; but in common with them the information is acquired by means of the angular distances between remarkable points on land as observed from the vessel, which should if possible be brought to anchor, or steadily hove to, while the observations are being made. But as the angles measured are subtended by objects on shore which are not and cannot be visited, arbitrary definitions must be employed to denote and recognise those objects, and sketches or profiles of the coast should be made from each point of observation. Indeed, under the present circumstances, such profiles may be said to be absolutely indispensable. These profiles save the necessity for written description, and assist in detecting angles that may have been entered glaringly wrong through haste or any other cause: they are also of great assistance in protracting the work, by bringing back a vivid recollection of the appearance of the points observed, their situation to the right or left of the station of departure, and other

circumstances attending the observation: finally, they are advantageously referred to when sketching in the ground-plan or contour of the coast. It is customary also to mark, on these profiles, each point observed by a distinctive letter, and to write opposite to it the angle which it forms with the station of departure, in addition of course to the regular entry in the field-book. As a general rule, the result of each day's observation should be protracted in the evening, when every occurrence is fresh in the observer's memory.

At each station of the vessel, astronomical observations are made to determine its position, at the same time that the angular distances between objects on shore are measured: two observers should be employed, in order that the observations may be made at the same moment of time, the latter observing also the angle formed between the sun and his first point of departure on shore, especially when the sun is just appearing, or is not much elevated above the horizon, as in this case the reduction to the horizontal angle may be omitted. The azimuthal bearing also of the first point from the vessel is to be taken by one or more azimuthal compasses, and the mean azimuthal angle serves to confirm its direction as obtained from the observed position of the sun, and to give it independently, if the sun has not been observed.

Azimuthal angles, thus taken, even under favourable circumstances, cannot be relied upon nearer than to two degrees or more, owing to the movement of the vessel, and the constant change in the variation of the compass. In a small boat, as the motion is much greater, the errors may be expected to increase also.

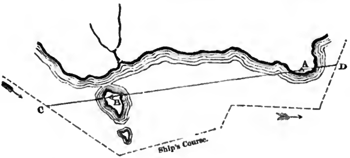
Similar observations bearing on the same and on new points that open in sight, are repeated at various distances as the vessel proceeds. In sailing from one vessel's station to another, especial care must be taken not to lose sight for a moment of the points to which angles have been


Station of
Departure.
Ships Course.

observed, as their continually changing aspects would otherwise make it difficult to recognise them. Inattention to this point will infallibly lead to numerous errors, and to delay and confusion in the construction of the charts. To assist the eye, a prominent aud easily recognised object should always be chosen as the point of departure.

Also while sailing from one ship's station to another, a reckoning of the rate of going is to be kept carefully by the log-line; this should seldom be relied upon to determine the ship's course, as its inaccuracy is notorious; but it may serve as a collateral check on the distances of the vessel's stations as obtained from astronomical observations, and must sometimes, per force, be used when angles are measured to objects on shore under conditions of the atmosphere that do not admit of astronomical observations.

When standing on and off the coast, especial care must be had to take advantage of the appearance, in the same straight line with the ship, of any two of the observed



points, the time of such transits being entered in the field-book. These bearings are useful when laying down the points and ship's course on the chart. It must be observed, however, that when one of the stations, thus appearing in the line, is very distant, there is no certainty of its extreme projection or lowest point being seen. This additional element of error may be avoided by having an observer on

the look out at the mast head, to give notice of the exact instant when he sees both the points in a line.

The angles measured are entered in the field-book, precisely in the same manner as those we have before described: the soundings and the reckoning, and other remarks, are entered in the field-book according to the form annexed.

H.	Knots	Fathoms.	Courses.	Winds.	Soundings.	Remarks.
					Fathoms.	
2	..	..	At anchor.	W.	15	Weighed, and stood to A.
3	1	6	S.S.W.	W.	14¼	
4	2	..	S.	.. ..	20	
5	2	..	S. by W.	.. ..	23½	
6	3	4	S.S.W.	.. ..	20	
7	2	..	S.W.	N.W.	18½	
8	$\frac{1}{1}$	..	E. by N.	Tacked.	22¾	
9	1	4	E.	W.	27	
10	1	6	S.W.	Tacked.	21	
11	3	..	.. ..	S.W.	19	
12	3	..	.. ..	S.W.	16½	

Measuring Distances by Sound.

Sound, as a means of ascertaining great distances approximatively, is not to be neglected. Its velocity, as ascertained by the latest experiments *, may be assumed at 1089, or in round numbers 1090 feet per second of time: a watch, therefore, by which an observer can measure with accuracy fractions of seconds, will enable him to determine a distance of several miles within about 100 or 200

* *Phil. Trans.*, 1824. Dr. Moll's Account of the Experiments on the Velocity of Sound.

yards. Observations of this nature should not be undertaken, without being provided with a good instrument for measuring fractions of a second with precision. A stop watch, known by the name of chronograph, answers this condition. It is so constructed, that one of its hands, which performs a revolution in a second, can be made to touch, with its extremity, the dial-plate at any instant of time, by the sudden pressure of a lever, and, leaving there a black dot, proceed without more than this momentary stoppage of its rotation.

To estimate distances by this method, two boats or vessels are moored at some distance from each other, and guns are fired alternately from each vessel, whilst the time elapsed between the flash and the report is noted by means of the stop watch. The time occupied by the passage of the light is equal to zero, and a simple proportion gives the distance between the two vessels. Angles being observed from each vessel to objects on shore previously agreed upon, the positions of these objects are determined with relation to the base or distance between the vessels.

Distances may also be obtained by approximation by means of the instrument known as Dr. Brewster's micrometer telescope, and described in BREWSTER's *Philosophical Instruments*.

Charts.

Charts are protracted with the true meridian pointing towards the top. At convenient places a mariner's compass is drawn, and the variation is shown by a small fleur-de-lis, which terminates the magnetic north and south line, drawn at the proper angle through the centre of the compass. Along the coast lines of 1 fathom, 2 fathoms, 3 fathoms, &c., are marked by dotted lines, thus,—

. . . .
.. .. } 1, 2, 3 fathoms lines.
... ...

The run of the flood tide is marked thus: →

That of the ebb tide thus: . . . . . . →

Buoys are marked thus . . . . . .

Good anchoring places . . . . . ⚓

Stopping places . . . . : .

Between high and low water: —

Rocks . . . .

Sand . . . .

Mud . . . .

Under the soundings, letters are added, denoting the nature of the bottom, thus:—

s, for sand;

m, for mud;

r, for rock; &c.

The measure of length employed is that of nautical miles, each mile being divided into 10 cables.

THE END.

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